Financial Innovation and Liquidity Premia in Sovereign Debt Markets

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Abstract

Issuances of state-contingent sovereign bonds have been limited both in quantity and frequency. One of the reasons argued in the literature is that these bonds would carry sizable liquidity premia given the smaller size of their market. This paper quantifies how liquidity premia erode the potential benefits of introducing new debt instruments. I incorporate search frictions into a standard incomplete-markets sovereign debt model. The model features free entry of dealers and an increasing-returns-to-scale matching technology, linking the liquidity of new debt instruments to the size of their secondary market. For the quantitative analysis, I focus on GDP-linked bonds. When the outstanding amount of GDP-linked bonds is small, search frictions are more severe, as only a few dealers enter the market. This, in turn, leads to higher bid-ask spreads and a larger liquidity premium at issuance, raising the government's financing costs. As a result, welfare gains are reduced by more than 50%.

Keywords: Sovereign debt, financial innovation, liquidity premium, GDP-linked bonds.
JEL codes: F34, G12, G15, D83

1 Introduction

Sovereign debt across emerging and developed countries is mainly composed of nominal bonds denominated in local or foreign currency. Apart from inflation-linked bonds, issuances of state-contingent debt instruments (SCDIs) have been limited both in quantity and frequency.[1] Why are sovereign governments reluctant to introduce new types of debt instruments? The existing literature has argued that the limited use of SCDIs can be explained by the sizable liquidity premium associated with new debt instruments given the smaller size of their market.[2] This is in fact what happened in the case of the US TIPS (Treasury Inflation Protected Securities). Their liquidity premium was around 100bps upon their introduction in 1997 and decreased to 30-50bps during the 2000s, as the market for these bonds increased.[3]

In this paper, I quantify the impact of liquidity premia on governments' incentives to introduce new types of sovereign debt instruments. I analyze a small open economy model with incomplete markets and limited commitment. The government issues long-term bonds in competitive international markets. It lacks commitment and it can default on its debt obligations. In the baseline case, the government can only issue non-contingent bonds. I then consider a scenario in which the government has the opportunity to start issuing new state-contingent debt instruments.

To model liquidity risk, I build on the search-based model of Duffie et al. (2005). In particular, I assume that a share of bondholders become liquidity constrained, prompting them to sell their holdings. I introduce search frictions into secondary markets by assuming that it takes time for an investor to find a counterparty (dealer) to sell their position. Waiting times are endogenous and depend negatively on market size. Since our focus is on the introduction of a new type of instrument, this negative relationship is central to the analysis. To create such a link, I allow for free entry of dealers, as in Lagos and Rocheteau (2009), together with an increasing-returns-to-scale matching technology. Dealers' benefits depend positively on the number of investors looking for a counterparty and on the bid-ask spread, which is determined through Nash bargaining. A

[1]For a comprehensive list of SCDIs issued by a sovereign government, see IMF (2017).

[2]See, among others, Kim and Ostry (2018), Borensztein et al. (2018), Sandleris and Wright (2014), and Borensztein and Mauro (2004).

[3]See Blanchard et al. (2016), Auckenthaler et al. (2015), D'Amico et al. (2014), Fleckenstein et al. (2014), Pflueger and Viceira (2013), Campbell et al. (2009), and Haubrich et al. (2006).

larger outstanding amount attracts more dealers and reduces search times. Shorter search times reduce the liquidity premium demanded by investors at issuance, which lowers the government's financing costs

To keep the model tractable, I only introduce search frictions in the market for contingent debt. Put it differently, the market for non-contingent bonds is assumed large enough so that search frictions and liquidity premia are negligible. The model can thus be viewed as one in which the secondary markets for non-contingent and contingent bonds are partially segmented. Otherwise, arbitrageurs would eliminate any abnormal spread differential across bond types (hence the liquidity premium). Partial segmentation may arise, as institutional or informational frictions lead intermediaries to specialize in a particular asset class or in a narrow set of assets (Greenwood et al., 2018 or Grossman and Miller, 1988). The assumption of partially segmented markets is consistent with the empirical evidence for the US TIPS market, since these bonds exhibit a positive liquidity premium. Krishnamurthy and Vissing-Jorgensen (2012; 2013), Gabaix et al. (2007), and Longstaff (2004) are examples of studies documenting some degree of market segmentation and limits to arbitrage across financial assets.[4]

To discipline the model, I focus on a particular type of state-contingent debt instruments: GDP-linked bonds. These bonds present appealing macro-prudential features that have been extensively discussed in the literature. First, by linking interest payments to GDP growth, they lower debt services during recessions, allowing to insure the economy against adverse shocks. Second, they reduce the likelihood of crises by keeping debt-to-GDP ratios at sustainable levels. Lastly, they can act as automatic stabilizers, since they allow for a smoother path for government spending, which reduces the need for procyclical policies.[5] Despite these benefits, GDP-linked bonds have only been issued by a small number of countries, mainly as part of a debt-restructuring episode.[6] While

[4] Krishnamurthy and Vissing-Jorgensen (2012) find evidence of market segmentation between US Treasuries and Aaa corporate bonds. Longstaff (2004) finds that US Treasury bonds trade at a premium relative to bonds issued by Refcorp (a US government agency whose bonds have an almost identical credit risk relative to Treasury bonds). More generally, by analyzing the effects of quantitative easing policies, Krishnamurthy and Vissing-Jorgensen (2013) find evidence indicating that the markets for mortgage backed securities (MBS), corporate bonds, and US Treasures are partially segmented from one another. Gabaix et al. (2007) also provide evidence supporting the existence of market segmentation in MBS markets.

[5] Procyclical fiscal policies are common among developing countries, in particular, in countries with higher sovereign risk (Bianchi et al., 2020, Arellano and Bai, 2017, Balke and Ravn, 2016, Cuadra et al., 2013, and Kaminsky et al., 2004).

[6] As stated by Shiller (2018), the absence of GDP-linked debt around the world is sort of a puzzle, given all the benefits associated with this type of debt. In the 1980s, Bulgaria, Bosnia and Herzegovina,

only GDP-linked bonds are considered, the model is flexible enough to accommodate other types of SCDIs, such as inflation-indexed bonds or bonds whose coupon payments are attached to the price of commodities. In this regard, the model can be interpreted as a general framework to study the limitations of financial innovation in sovereign debt markets.

I calibrate the model for an economy in which the government only issues non-contingent bonds. I then quantify the welfare gains of an unanticipated (credible) announcement stating that, from that time onwards, the government can also issue state-contingent bonds up to a certain limit. This type of analysis allows to characterize the optimal way of introducing new types of financial instruments once search frictions are accounted for.

The results can be summarized as follows. In a model without search frictions, welfare gains of issuing GDP-indexed debt are in the order of 0.07 to 0.22 percent in terms of certainty equivalent (CE) consumption (depending on the initial state and on the announced debt limit). The largest welfare gains are typically obtained when the economy is in a recession, since this type of debt allows for smaller interest payments until the economy recovers.[7] Once search frictions are considered, welfare gains of introducing GDP-linked bonds are reduced by more than 50%. The larger decrease is observed when the government cannot commit to issue a large amount of indexed bonds. The smaller welfare gains are surprising, since the calibration of the model yields a conservative liquidity premium of only 35 basis points, which can be considered as a lower bound. [8]

The model also sheds light on the optimal way to introduce new types of debt instruments and on the composition of the optimal debt portfolio. For the case with no-search frictions, the government issues indexed bonds at once. Immediately after the announcement, there is a sharp increase in the amount issued of such bonds (almost up to the announced limit) and a large buy-back of non-indexed debt. For the case with search frictions, the government finds it optimal to introduce these bonds at a slower pace.

and Costa Rica issued GDP-linked bonds as part of the Brady restructuring. More recently, Argentina (2005) and Greece (2012) issued these bonds also as part of a debt-restructuring process.

[7]In a recession, the price of GDP-linked bonds is lower than the price of non-contingent bonds (given the smaller interest payments), implying a larger financing cost for the government. However, this larger cost is more than compensated by the lower interest payments. This follows from the fact that the sovereign is risk averse, investors are risk neutral, and the time it takes until the economy recovers is stochastic.

[8]A liquidity premium of 35 bps is the premium associated with the US TIPS, arguably one of the most liquid indexed bonds.

There are two opposite forces behind this last result. On the one hand, a larger issuance of indexed bonds decreases the liquidity premium, which provides incentives to issue a large amount at once. On the other hand, due to the liquidity premium, the price of non-indexed bonds is (typically) higher than the price of indexed debt. Issuances of indexed-bonds thus lead to an increase in the total stock of debt, since the government is unable to buy back the same amount of non-indexed bonds. The larger stock of debt represents a cost for the government, since it implies lower consumption in the future. In the quantitative analysis, I show that the latter effect is larger and, therefore, the government optimally chooses to introduce indexed bonds at a slower pace.

Finally, I consider a model extension that features: (i) a continuum of hand-to-mouth domestic households that are heterogeneous in their income, and (ii) certain limitations on the government's capacity to redistribute resources across the households. Following Ferriere (2015), I assume that the government faces a tax function that is linear in the households' income and that it cannot change its progressivity. Under this extension, welfare gains of introducing GDP-linked bonds are 30 to 60% higher. The intuition behind this result is that GDP-linked bonds decrease the need for procyclical fiscal policies, which are particularly costly in this context given its disproportionate effect on low income households. The key message from this extension is that those countries with higher income inequality and lower tax progressivity are the ones that can benefit the most from issuing SCDIs. However, given that these features are typically associated with less developed countries, the larger benefits may be attenuated by the larger frictions in their secondary markets, which leads to an increase in the liquidity premium at issuance.

Related Literature. This paper builds upon the literature on financial innovation in sovereign bond markets. It also relates to a broader literature that analyzes how search frictions in over-the-counter markets affect the spreads of debt instruments.

The current study contributes to the literature on the lack of financial innovation in sovereign debt markets. Apart from the liquidity premium motive, there are other reasons argued in the literature. A first argument is related to moral hazard concerns. Since many SCDIs are indexed to variables under the control of the government, the government may have incentives to tamper with the underlying index to decrease its interest payments (Morelli and Moretti, 2021).[9] Anticipating this behavior, investors

[9]To ameliorate these concerns, the literature has also considered SCDIs indexed to real variables that are not under the direct influence of the government. For instance, Krugman (1988), Froot et al. (1989),

may demand a premium for buying these bonds, increasing the costs of issuance.[10] A second explanation is related to adverse selection. For SCDIs, the underlying variables to which these bonds are indexed may not be perfectly observable, generating information asymmetries between the government and private investors. Glosten and Milgrom (1985), Back and Baruch (2004), and Lester et al. (2018) analyze how information asymmetries regarding the fundamental value of an asset affect the bid-ask spread in the secondary market of that security. Lastly, Roldán and Roch (2020) show that model misspecification on the part of lenders can increase the ambiguity premia of SCDIs, reducing the benefits of introducing such bonds. While I do not consider these other motives, this paper is the first to quantify the impact of liquidity premia on the benefits of introducing new types of sovereign debt instruments.

The paper is closely related to the theoretical literature that analyzes how search frictions in the secondary market of a particular security affect its spreads. Using the framework by Duffie et al. (2005), He and Milbradt (2014) and Chen et al. (2017) analyze the interaction between default and liquidity risk for corporate US bonds traded in OTC markets. In terms of sovereign debt markets, two closely related studies are Passadore and Xu (2020) and Chaumont (2020). These papers focus on the interaction between sovereign default and liquidity risk in a general equilibrium model in the spirit of Eaton and Gersovitz (1981) and Arellano (2008). To model liquidity risk, Chaumont (2020) follows a competitive-search framework in which investors direct their search to different submarkets. Passadore and Xu (2020), on the other hand, formulate a random-search type of model, as in Duffie et al. (2005). The main difference with my paper is that, in Passadore and Xu (2020), trading frictions are exogenously imposed by assuming a fixed contact probability between investors and dealers. In this paper, I endogenize this probability based on the size of the secondary market, which is a necessary ingredient to study the introduction of a new type of debt instrument.

More generally, this paper adds to the literature on the endogenous determination of the supply of assets with different degrees of liquidity. Overall, the model presented in this paper quantitatively characterizes the endogenous supply of liquid (non-indexed

and Caballero (2002) consider debt instruments indexed to the terms of trade and commodity prices.

[10] Moral hazard concerns are not exclusive of indexed bonds. Perez and Ottonello (2019) for instance, formulate a model in which the government can dilute its stock of local-denominated nominal bonds through (unexpected) currency depreciation. This lack of commitment increases the spreads on local-denominated bonds and the government finds optimal to tilt its debt toward foreign-denominated bonds.

bonds) and illiquid (indexed bonds) assets. Geromichalos and Herrenbrueck (2020) also develop a theory on the endogenous determination of the supply of liquid assets, but in a context of strategic interaction among asset issuers. This paper differs in two aspects. First, assets are subject to default risk. Second, there is no strategic interaction, since there is only one issuer (the government).

Lastly, the paper expands on the literature that quantifies the benefits of introducing new types of sovereign debt instruments. For the context of a small-open economy with incomplete markets and limited commitment, Faria (2007), Sandleris et al. (2011), Hatchondo and Martinez (2012), and Onder (2017) quantify the welfare gains of issuing GDP-indexed bonds.[11] In related work, Hatchondo et al. (2016a) show that bonds that automatically extend in repayment maturity when the country is in distress can reduce the frequency of sovereign defaults and increase welfare.[12] I extend this strand of the literature by quantifying how these welfare gains change once the liquidity premium for these bonds is considered. Moreover, I allow the government to issue both non-indexed and indexed debt, which allows me to characterize the optimal debt portfolio.

The paper is organized as follows. Section 2 describes the model. Section 3 details the calibration. Section 4 presents the quantitative analysis. Section 5 describes an extension of the model with heterogeneous households and limited tax progressivity. Section 6 concludes.

2 The Model

I consider a small open economy (SOE) model with incomplete markets, limited commitment, and exogenous cost of default in which the sovereign government can issue long-term non–indexed bonds and state-contingent bonds. The government is benevolent and maximizes the utility of the representative household by issuing external debt. Its preferences are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{2.1}$$

[11] These studies find that the welfare gains derived from issuing such bonds are in the order of $0.1-0.5$ percent in terms of certainty equivalent consumption. Durdu (2009) also analyzes the benefits of introducing these bonds but abstracting from limited commitment and default decisions on the part of the sovereign.

[12] These bonds are usually referred to as Sovereign Cocos (contingent convertible) and were first discussed by Weber et al. (2011) and Brooke et al. (2013).

where β is the discount factor, c_t represents aggregate consumption, and the utility function $u(.)$ is strictly increasing and concave. Given an exogenous and stochastic Markov process for endowment (y), the government chooses both long-term non-indexed bonds (b) and long-term state-contingent (i.e., indexed) bonds (B), as well as defaults, in order to maximize (2.1).

Following Chatterjee and Eyigungor (2012), I consider long-term debt contracts that mature probabilistically.[13] In particular, a unit of non-indexed (indexed) debt matures next period with probability m_b (m_B). If the non-indexed bond does not mature (and the government does not default), it gives a (constant) coupon payment of z_b. For the case of state-contingent bonds, the coupon payment is given by: $z_B(\boldsymbol{S}_t)$, where $\boldsymbol{S}_t$ denotes the aggregate state of the economy.[14]. Based on these assumptions, if the country is not currently in default, its resource constraint is given by:

$$\begin{aligned} c_t = y_t - b_t\left[(1-m_b)\, z_b + m_b)\right] + q_t^{ND}\left[b_{t+1} - (1-m_b)b_t\right] + \\ - B_t\left[(1-m_B)\, z_B(\boldsymbol{S}_t) + m_B)\right] + p_{U,t}^{ND}\left[B_{t+1} - (1-m_B)B_t\right] \end{aligned} \tag{2.2}$$

where q_t^{ND} is the price of non-indexed bonds (when the country is not in default), and $p_{U,t}^{ND}$ is the price (at issuance) of indexed bonds.

At issuance, both types of bonds are priced in a competitive market composed of a large mass of risk-neutral foreign investors. Their objective is to maximize the present value of a stream of payoffs and they do not have a particular preference for one asset or another.

If the sovereign government defaults, the country is excluded from international markets for a stochastic number of periods. While excluded from markets, consumption is given by $y_t - \phi(y_t)$, where $\phi(y_t)$ is an exogenous output cost of being excluded. With exogenous probability θ the country exits default and regains access to international markets. Finally, a debt recovery rate f after exiting the default is assumed.[15]

To keep the model tractable, it is assumed that only state-contingent bonds are subject to liquidity risk. This assumption can be rationalized by the fact that the secondary market for non-indexed bonds is significantly larger. Given the larger size, there are more

[13]Long-term debt is needed so that investors are exposed to liquidity risk.
[14]To simplify the analysis, the principal of the bond is not indexed to any aggregate variable.
[15]A positive recovery value is needed in order to generate positive bid-ask spreads during a default episode.

Figure 2.1: Timing of Events within each Period

t $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $t+1$

Government starts with $\bar{\mathbb{S}}$ and chooses to default or not.	If not in default, government chooses (b', B')	Dealers enter the market	Trades in the secondary market for state-contingent bonds occur	Investors holding a state-contingent bond may suffer a liquidity shock

intermediaries trading these bonds, so it takes a short fraction of time to effortlessly find a counterparty to sell the asset.

To model liquidity risk, I build on the random over-the-counter (OTC) search framework by Duffie et al. (2005). It is assumed that those investors who are holding one unit of an indexed bond are subject to uninsurable liquidity shocks. Every period, a fraction ζ of these investors become liquidity constrained, in which case they will try to sell their bond in the secondary market. To sell their position, constrained investors have to meet first with a dealer. The rate at which they meet, λ_t, is endogenous and depends on the measure of constrained investors that are trying to sell (η_t). To create such a link, I allow for free entry of dealers, as in Lagos and Rocheteau (2009), together with an increasing-returns-to-scale matching technology.

From the point of view of a government that is currently out of default, the timing of events is as follows:

1. Government starts with current state $\boldsymbol{S} = (y, b, B, \eta)$ and chooses to default or not.
2. If not in default, government chooses (b', B') taking as given the price schedules $q^{ND}(.)$ and $p_U^{ND}(.)$.
3. Dealers decide to enter the market.
4. Trades in the secondary market for state-contingent bonds occur.
5. Unconstrained investors holding one unit of a state-contingent bond may suffer a liquidity shock, in which case they will be liquidity constrained next period.

2.1 Description of the Search Frictions: The Secondary Market for state-contingent bonds

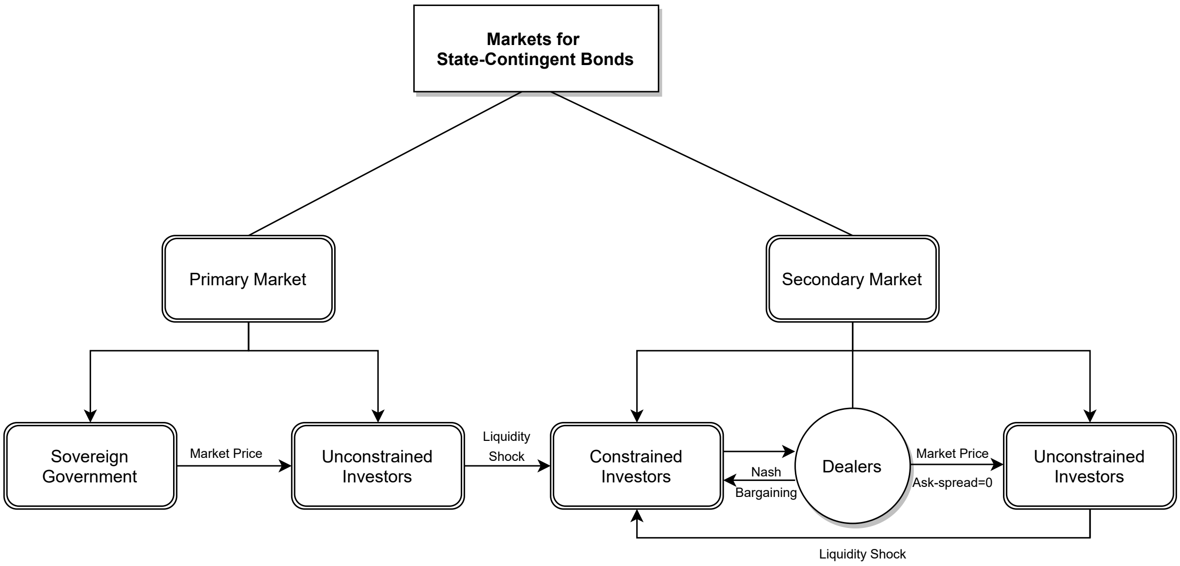
Search frictions for state-contingent debt are introduced in the same way as in Passadore and Xu (2020), which is based in Duffie et al. (2005) -DGP hereafter- and in He and Milbradt (2014). In particular, there are two types of investors for state-contingent bonds: unconstrained and liquidity constrained investors. There is a large mass of unconstrained investors, who discount payoffs at rate r_U and participate in the primary market at the time of issuance and as buyers in the secondary market. With probability ζ an unconstrained investor becomes liquidity constrained and discounts payoffs at rate $r_C > r_U$.[16] As in He and Milbradt (2014), it is assumed that liquidity constrained investors become unconstrained with probability zero. This implies that once an investor that is holding an indexed bond is hit by a liquidity shock, that investor will always like to sell the asset in the secondary market. Let η denote the measure of constrained investors.

As in DGP (2005), it takes time for a liquidity constrained investor to find a counterparty (dealer) to sell its current position. In Passadore and Xu (2020), this is captured by assuming that constrained investors meet dealers with exogenous (constant) probability λ. I extend this set-up by endogenizing this probability so that it depends on the measure of liquidity constrained investors, η. In other words, waiting times depend on the size of the secondary market. To create such an endogenous link, I allow for free entry of dealers, following Lagos and Rocheteau (2009). In particular, I assume that dealers have to pay a flow cost $\kappa > 0$ in order to trade state-contingent bonds bonds. On the other hand, dealers' benefits depend both on the number of constrained investors participating in the secondary market (η), and on the bid-ask spread, which is determined through Nash bargaining and it will be described below.

Figure 2.2 summarizes the two markets for state-contingent bonds. The primary market is composed of the sovereign government and risk-neutral foreign unconstrained investors. In this market, the government issues bonds and the unconstrained investors buy them at market price. The secondary market is formed by dealers, constrained investors (sellers), and unconstrained buyers. It is assumed that whenever a constrained

[16] The rise in the discount rate captures the idea of "urgency to sell". He and Milbradt (2014) show that results are qualitative the same if liquidity shocks are modeled by a rise of the discount rate or by a holding cost proportional to both the coupon and principal of the bond.

Figure 2.2: State-Contingent Bonds: Primary and Secondary Markets



Notes: Figure describes the primary and secondary markets for state-contingent (indexed) bonds.

investor meets a dealer, they negotiate the bid price through Nash bargaining. On the other hand, I assume that dealers sell to unconstrained investors at market price, and therefore they obtain no profit from this transaction. In other words, after a dealer has bought the asset from the constrained seller, I assume that the dealer can sell it (without search frictions) to an unconstrained buyer. This assumption is for simplicity as it allows not to keep track of both constrained and unconstrained investors. The assumption can be rationalized by the observation that, to the extent that the government is currently issuing a state-contingent bond bonds, an unconstrained investor should be indifferent between buying it in the primary or secondary market. Hence, the price at which dealers sell this asset in the secondary market should be equal to the price in the primary market. Finally, direct trades between investors are not considered.

2.1.1 Bid Prices in the Secondary Market - Nash Bargaining

This subsection describes the Nash bargaining that determines the bid price in the secondary market. Transactions in the secondary market can occur both when the country is out or in default. As the two cases are analogous, in this section I focus only on the case in which the government is not in default.

Consider a liquidity constrained investor that has met a dealer and is willing to sell a unit of a state-contingent bond. The dealers' surplus is given by the difference between the price at which they can sell the bond to an unconstrained investor, $p_U^{ND}(\mathbb{S})$, and the bid price $p_B^{ND}(\mathbb{S})$, where $\mathbb{S} \equiv (y, b', B', \eta)$ denotes the state vector after the new bond issuances. For the constrained investor, the surplus of the transaction is given by the difference between the bid price and what the constrained investor values the state-contingent bond given that the sovereign government is out of default, $p_C^{ND}(\mathbb{S})$ (defined in the next section). Let α be the bargaining power of the constrained sellers. The bid price is the solution to the following Nash bargaining problem:

$$
\begin{gathered}
p_B^{ND}(\mathbb{S}) = Argmax_{p^{ND}(\mathbb{S})} \left(p^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right)^{\alpha} \left(p_U^{ND}(\mathbb{S}) - p^{ND}(\mathbb{S})\right)^{1-\alpha} \\
s.t.\ p^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S}) \geq 0 \\
p_U^{ND}(\mathbb{S}) - p^{ND}(\mathbb{S}) \geq 0
\end{gathered}
$$

From the first order condition, the bid price is given by: $p_B^{ND}(\mathbb{S}) = (1-\alpha)\, p_C^{ND}(\mathbb{S}) + \alpha p_U^{ND}(\mathbb{S})$. Hence, conditional on finding a constrained seller to trade with, dealers' profits from each transaction (that is, the spreads in the secondary market) are given by: $p_U^{ND}(\mathbb{S}) - p_B^{ND}(\mathbb{S}) = (1-\alpha)\left(p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right)$. As it will be clear once we define the dealers' entry problem and the sovereign government's problem, $p_C^{ND}(\mathbb{S})$ depends positively on the probability of finding a dealer, which is an increasing function of the number of state-contingent bonds outstanding. Thus, the spreads in the secondary market are decreasing in the stock of state-contingent bonds.

2.1.2 Dealers' Entry Problem

I allow for free entry of dealers in order to endogenize both the measure of active dealers and the length of the trading delays in the secondary market for state-contingent bonds. As in Lagos and Rocheteau (2009), free entry of dealers also allows to capture the notion that a dealer's profit depends on the competition for order flow that it faces from other dealers. It is assumed that a large measure of dealers can choose to participate in the market and, while they participate, they incur a flow cost κ, which captures the ongoing costs of running the dealership. In a sense, κ captures the costs of advertising their services to investors, costs relative to obtaining information about the bonds that they trade, maintaining access to inter-dealer market, among others. Let v denote the measure of active dealers.

Let η be the measure of constrained investors (sellers) that are holding a state-contingent bond and would like to sell it. It can be shown (see Appendix A) that η satisfies the following recursion:

$$\eta' = \eta (1 - m_B) (1 - \lambda (v, \eta) - \zeta) + B' \zeta \tag{2.3}$$

Intuitively, the first term on the right-hand side of equation (2.3) captures those investors that were already liquidity constrained at the beginning of the period, whose debt did not mature, and that were not able to find a dealer to sell their position. The second term on the right-hand side involves those investors that were not liquidity constrained at the beginning of the period (denoted by $\tilde{\eta}$), whose debt did not mature, and that were subject to a liquidity shock at the end of the period. The last term represents those investors that, at the beginning of the period, bought a bond in the primary market, this bond did not mature and they suffered from a liquidity shock at the end of the period.[17]

I assume that the total flow of matches between dealers and investors is given by the matching function $m (v, \eta)$, which is strictly increasing in both arguments, concave, and displays increasing returns to scale (IRS). The latter assumption seems realistic for financial markets because it implies that investors can find a trading partner more easily in larger markets. Moreover, it fits the well-documented fact that trading costs are decreasing with trading volume. Vayanos and Wang (2007) and Geromichalos and Herrenbrueck (2020) are recent examples that also use an IRS matching technology when modeling over-the-counter (OTC) markets. Appendix B provides micro-foundations for the use of an increasing-returns-to-scale matching technology, based on a model of random search. Appendix E briefly analyzes the constant-returns-to-scale case. As each investor is equally likely to find a dealer, the probability for an investor to find a counter-party to trade with is given by: $\lambda (v, \eta) \equiv \frac{m(v,\eta)}{\eta}$. Similarly, once they enter the market, all dealers are the same, and therefore the probability for a dealer to find an investor is given by: $\lambda^v (v, \eta) \equiv \frac{m(v,\eta)}{v}$.[18] The imposed assumptions on $m (v, \eta)$ imply that $\lambda (v, \eta)$

[17]Remember that the timing assumption is that government's issuances take place at the beginning of the period while trades in the secondary market take place at the end of the period, before the realization of the liquidity shock ζ. Therefore, an investor that suffered from a liquidity shock in period t cannot find a dealer to trade with during period t.

[18]Previous notation corresponds to the case in which the government is currently not in default. If the government is in default, these probabilities are denoted as $\lambda_D (v, \eta)$ and $\lambda_D^v(v, \eta)$, respectively.

is continuous and strictly increasing in v. On the other hand, I assume that $\lambda^v(v,\eta)$ is strictly decreasing in v, which captures the notion of competition for order flows as in Lagos and Rocheteau (2009). Assuming that the government is not currently in default, dealers' free-entry condition implies:[19]

$$\Pi(\mathbb{S};v) \equiv \lambda^v(v,\eta) \times (1-\alpha)\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right] - \kappa = 0 \tag{2.4}$$

Consider a given number of dealers $v^\star$, such that $\Pi(\mathbb{S}, v^\star) = 0$. Then, notice from equation (2.3) that an increase in the stock of state-contingent bonds (B') leads to an increase in tomorrow's measure of constrained investors (η'). Denote this state as $\tilde{\mathbb{S}}$. For the sake of intuition, assume that this increase is such that: $\Delta\left(p_U^{ND}\left(\tilde{\mathbb{S}}\right) - p_C^{ND}\left(\tilde{\mathbb{S}}\right)\right) \approx 0$. For a given $v = v^\star$, the increase in the measure of constrained investors leads to an increase in $\lambda^v(.)$ and therefore $\Pi\left(\tilde{\mathbb{S}}, v^\star\right) > 0$. The free-entry condition on equation (2.4) will therefore imply that more dealers enter the market, which (as it will be shown in the quantitative exercise) leads to shorter trading delays and lower bid-ask spreads in the secondary market. Shorter trading delays and lower bid-ask spreads, in turn, lead investors to demand a lower liquidity premium for buying state-contingent bonds, decreasing the financing costs of the sovereign government.

2.2 Sovereign's Government Recursive Problem

The government is benevolent and seeks to maximize (2.1) subject to the resource constraint (2.2), by choosing consumption c, the optimal portfolio composition (b', B'), and optimal default decisions. Recursively, the problem can be stated as:

$$V(y,b,B,\eta) = Max_{d\{0,1\}}\left\{V^d(y,b,B,\eta), V^r(y,b,B,\eta)\right\} \tag{2.5}$$

[19] The free-entry condition is analogous for the case in which the government is in default.

where $V^r(.)$ is the value of repayment and is given by:

$$V^r(y,b,B,\eta) = Max_{c,b'\geq 0,B'\geq 0}\ u(c) + \beta \int_{y'} V(y',b',B',\eta')\, dF(y' \mid y) \tag{2.6}$$

$$\begin{aligned} s.t.\ c = y - b\left[(1-m_b)\, z_b + m_b)\right] + q^{ND}(y,b',B',\eta)\left[b' - (1-m_b)b\right] + \\ - B\left[(1-m_B)\, z_B(\boldsymbol{S}) + m_B)\right] + p_U^{ND}(y,b',B',\eta)\left[B' - (1-m_B)B\right] \\ \left[b' - (1-m_b)\, b\right] + \left[B' - (1-m_b)\, B\right] > 0 \text{ only if } q^{ND}(y,b',B',\eta) \geq \underline{q} \\ \eta' = \eta\,(1-m_B)\,(1-\lambda(v,\eta)-\zeta) + B'\zeta \\ (1-m_B)\, B \leq B' \leq \bar{B} \end{aligned}$$

where v is such that the free-entry condition on equation (2.4) is satisfied.

A known problem in models with long-term debt is that introducing a positive recovery rate ($f > 0$) increases the volatility of the sovereign spread. This is explained by the fact that, for the period before defaulting, the government may find optimal to issue an infinite amount of debt that fully dilutes the value of previous debt claims and that allows for a consumption boom. To overcome this issue, I follow the methodology adopted by Hatchondo et al. (2016b) and assume that the government cannot issue bonds if that issuance implies a price lower than some threshold $\underline{q}$, for non-indexed bonds.

The last constraint in (2.6) adds the restriction $B' \leq \bar{B}$, imposing a limit to the maximum amount of state-contingent bonds that the country can issue. Without this limit and in the absence of search frictions, the sovereign government would end up (in the long-term) with indexed debt only, which is counterfactual. This methodology is also used by Hatchondo et al. (2017) in their paper of defaultable and non-defaultable debt. For instance, these authors chose a 10% limit (in terms of annual GDP) for the stock of non-defaultable debt as they argue that: (i) this limit is at the lower end of limits discussed in Euro-bond proposals regarding non-defaultable debt, and (ii) even with this limit they can generate non-negligible welfare gains and changes in spreads. For this paper, I allow for two different debt limits regarding the stock of state-contingent bonds: a 10% and a 25% limit.

The government could in principle repurchase state-contingent bonds held by constrained investors at a price that is smaller than p_U^{ND}, sell them again to unconstrained investors, and obtain a profit for that transaction. Under this scenario, moreover, constrained investors would need to decide whether to sell the bond to the government or to wait and find a dealer to trade it with. To keep the model tractable, I rule out the possibility of buy-backs and impose $B' \geq (1-m_B)\,B$.[20,21]

The sovereign's value of default is given by:[22]

$$V^d\left(y,b,B,\eta\right) = \; u\left(y-\phi(y)\right) + \beta\times\theta\times\int_{y'} V\left(y',f\times b,f\times B,f\times\eta\right)dF\left(y'\mid y\right) + \beta\times\left[1-\theta\right]\times\int_{y'} V^d\left(y',b,B,\eta'\right)dF\left(y'\mid y\right) \tag{2.7}$$

where $\phi\left(y\right)$ is the exogenous cost of default and θ is the probability of regaining access to international markets. To simplify notation, let $h\equiv h\left(y,b,B,\eta\right)$ denote the government's optimal default decision given the current state.

2.2.1 Bond Prices

I describe next the pricing kernels for both non-indexed and indexed bonds. As non-indexed bonds are not subject to search frictions, their pricing is simply given by:[23]

$$q^{ND}\left(y,b',B',\eta\right)\left[1+r_U\right] = \int\left\{\left[1-h'\right]\left[m_b+\left(1-m_b\right)\left[z_b+q^{ND}\left(y',b'',B'',\eta'\right)\right]\right] + h'\left[q^D\left(y',b',B',\eta'\right)\right]\right\}dF\left(y'\mid y\right) \tag{2.8}$$

$$q^D\left(y,b,B,\eta\right) = \frac{1-\theta}{1+r_U}\int q^D(y',b,B,\eta')dF\left(y'\mid y\right) + \theta\times f\times q^{ND}\left(y,f\times b,f\times B,f\times\eta\right) \tag{2.9}$$

[20]The assumption, moreover, reduces the computational time to solve for the optimal portfolio problem (b',B') as it allows to reduce the number of grid points for B. See Appendix C for details.

[21]Regarding GDP-linked bonds in particular, concerns have been raised regarding the *callability* of these bonds. If the government has the ability to buy back these bonds *at par*, then in a scenario in which the country is expected to grow more than what was originally expected (and therefore priced), the government can deprive foreign investors of these additional benefits by buying back the bonds. This is in fact what happened in Bulgaria, where the government decided to buy back these bonds when growth exceeded the nominated threshold rather than pay an additional premium (Griffith-Jones and Sharma, 2006 and Sandleris et al., 2011).

[22]The assumption is that the measure of constrained investors after exiting a default is proportional to this measure during the default. That is, during default, the stock of state-contingent bond and the measure of constrained investors is (B,η); after exiting default, these two values are given by: $(f\times B,f\times\eta)$. Moreover, η' refers to the evolution of the measured of constrained dealers if the country is in default. It is given by: $\eta'=\eta\left(1-\lambda_D(v,\eta)-\zeta\right)+B\zeta$.

[23]It is important to notice that the evolution of η depends on whether the country is in default (or not) today and whether the country is in default tomorrow. For easiness of notation, I simply write η' as the next-period value of η, irrespective of the current state.

Due to the presence of search frictions, the pricing kernels of indexed bonds are more involved. First, assuming that the government is not in default, the price of a state-contingent bond in the primary market is given by:

$$\begin{aligned}
&p_U^{ND}(y,b',B',\eta)\left[1+r_U\right] = \\
&\quad\int\Bigg\{\left[1-h'\right]\left[m_B+(1-m_B)\left[z_B(\boldsymbol{S'})+\zeta p_C^{ND}(y',b'',B'',\eta')+(1-\zeta)p_U^{ND}(y',b'',B'',\eta')\right]\right] \\
&\qquad\qquad + h'\left[\zeta p_C^D(y',b',B',\eta')+(1-\zeta)\,p_U^D(y',b',B',\eta')\right]\Bigg\}dF(y'\mid y) \qquad (2.10)
\end{aligned}$$

where $p_C^{ND}(.)$ represents the valuation of a constrained investor for a state-contingent bond. It is given by:

$$\begin{aligned}
&p_C^{ND}(y,b',B',\eta)\left[1+r_C\right] = \\
&\int\Bigg\{\left[1-h'\right]\left[m_B+(1-m_B)\left[z_B(\boldsymbol{S'})+\left[1-\lambda(v',\eta')\right]p_C^{ND}(y',b'',B'',\eta')+\lambda(v',\eta')\,p_B^{ND}(y',b'',B'',\eta')\right]\right] \\
&\qquad + h'\left[\left[1-\lambda_D(v',\eta')\right]p_C^D(y',b',B',\eta')+\lambda_D(v',\eta')\,p_B^D(y',b',B',\eta')\right]\Bigg\}dF(y'\mid y) \qquad (2.11)
\end{aligned}$$

where $p_B^i(.)$ for $i=\{D,ND\}$ are the outcomes of the Nash bargaining between the constrained investors and the dealers.

Finally, if the country is already in default, the pricing kernel of state-contingent bond is given by:

$$\begin{aligned}
p_U^D(y,b,B,\eta) = {}& \frac{1-\theta}{1+r_U}\int\left\{\zeta p_C^D(y',b,B,\eta')+(1-\zeta)p_U^D(y',b,B,\eta')\right\}dF(y'\mid y)+ \\
&+\theta\times f\times p_U^{ND}(y,f\times b,f\times B,f\times\eta) \qquad (2.12)
\end{aligned}$$

$$\begin{aligned}
&p_C^D(y,b,B,\eta) = \\
&\quad\frac{1-\theta}{1+r_C}\int\left\{\left[1-\lambda_D(v',\eta')\right]p_C^D(y',b,B,\eta')+\lambda_D(v',\eta')\,p_B^D(y',b,B,\eta')\right\}dF(y'\mid y)+ \\
&\qquad +\theta\times f\times p_C^{ND}(y,f\times b,f\times B,f\times\eta) \qquad (2.13)
\end{aligned}$$

2.3 Markov Perfect Equilibrium

A Markov Perfect Equilibrium is a set of value functions: $V^r(y,b,B,\eta)$ and $V^d(y,b,B,\eta)$; a set of policy functions for consumption, debt, and default: $c(y,b,B,\eta)$, $b'(y,b,B,\eta)$, $B'(y,b,B,\eta)$, and $h(y,b,B,\eta)$; and pricing kernels: $q^i(y,b,B,\eta)$, $p_U^i(y,b,B,\eta)$, and

$p_C^i(y,b,B,\eta)$ (for $i=\{D,ND\}$), such that:

1. Taking as given the price schedules $q^i(y,b,B,\eta)$ and $p_U^i(y,b,B,\eta)$ for $i=\{D,ND\}$, the government's policy functions $\{c(.),b'(.),B'(.),h(.)\}$ solve the government's problem in equations (2.5)-(2.7), and $V^r(.)$ and $V^d(.)$ are the associated value functions.
2. Taking as given the policy rules $\{c(.),b'(.),B'(.),h(.)\}$, the kernels $q^i(y,b,B,\eta)$, $p_U^i(y,b,B,\eta)$, and $p_C^i(y,b,B,\eta)$ satisfy equations (2.8)-(2.13) for $i=\{D,ND\}$.
3. Taking as given the price schedules $p_C^i(y,b,B,\eta)$ and $p_U^i(y,b,B,\eta)$ for $i=\{D,ND\}$, the number of dealers v is such that equation (2.4) holds.

3 Calibration

I use Spain to discipline the parameter values corresponding to the sovereign country. Spain's external debt has been steadily increasing during the last decade and concerns have been raised about the sustainability of such debt levels.[24]. This makes Spain an interesting case of study to explore the benefits of introducing new types of state-contingent debt instruments.[25] To simplify the analysis, I assume that the government can only issue one type of state-contingent bonds: GDP-indexed bonds. The calibration is done for the model in which the government cannot issue GDP-linked bonds ($\bar{B}=0$) and is described in Table 3.1.

3.1 Government's Problem Parameters

The utility function is a standard CRRA type:

$$u(c)=\frac{c^{1-\gamma}}{1-\gamma} \tag{3.1}$$

where $\gamma\neq 1$ is the relative risk aversion parameter. I assume that the relative risk aversion of the representative agent is 2 and I assume a discount factor of 0.98, which are standard values for models of sovereign default. The (quarterly) log-endowment process

[24] See, for instance, IMF, 2015.

[25] In addition, unlike Greece or Argentina, countries in which their governments have misreported macroeconomic variables, moral hazard may be a less important concern for the Spanish case.

follows an AR(1) given by:

$$log\left(y_t\right) = \rho\, log\left(y_{t-1}\right) + \epsilon_t;\ \epsilon_t \sim N(0, \sigma) \tag{3.2}$$

For a model of debt dilution and sovereign default risk, Hatchondo et al. (2016b) estimate equation (3.2) using quarterly real GDP data from Spain from 1960.Q1 to 2013.Q1. For the current version of this paper, I use their estimates for ρ and σ.

The parameter m_b is chosen to match an average maturity of 11 years, consistent with the average maturity of Spanish T-Bonds.[26] Moreover, given m_b, the coupon payment z_b is calibrated to match annual debt services. Using data from the European Central Bank (ECB), Spain total debt services due in 2 years were around 37% of the total stock of debt during 2009-2015. Therefore, given the value of m_b and the quarterly frequency of the model, I calibrate z_b so that debt services as a share of the stock of debt are: $m_b + (1 - m_b)\, z_b = \frac{0.37}{8}$, implying $z_b \approx 0.02$.[27]

For the case of state-contingent bonds, I assume that the government can only issue GDP-linked bonds. Their coupon payments are given by:

$$\begin{aligned} z_B\left(\boldsymbol{S}\right) &= z_B(y) \\ &= z_b + \epsilon\left(y_t - \bar{y}\right) \end{aligned} \tag{3.3}$$

where $\bar{y}$ is the (unconditional) mean of the endowment process and $\epsilon > 0$. Notice that there is no floor or ceiling and thus investors fully participate both to positive and adverse income realizations.[28] According to Onder (2017), this specification is the one associated with the largest welfare gains. The GDP-indexed bond described in this paper is what has been called a “floater”, as it is a bond with fixed principal and coupon linked to changes in a state variable.[29]

[26] This was the average maturity for Spanish T-Bonds with maturities from 3 to 50 years during 2009-2015. Average maturity is computed using as weights the amount outstanding of each type of bond. As a comparison, Hatchondo et al. (2016b) and Hatchondo et al. (2017) target an average duration (not maturity) of 6 years.

[27] Chatterjee and Eyigungor (2012) calibrate z_b so that the bonds would trade roughly at par, in order to avoid concerns regarding whether the debt is recorded at face value (accounting standard) or at market prices. Under the current calibration of $z_b = 0.02$, the average price of the non-indexed bonds is 1.09 in the simulations (excluding default episodes), so the bonds are also roughly trading at par.

[28] The parameter ϵ is calibrated in a way that guarantees: $z_B\left(y_t\right) > 0$ for all y_t.

[29] I consider this type of contract for simplicity, as I don’t need to keep track of other state variables other than y. Moreover, a recent survey taken by the IMF to potential buyers of this type of bonds remarks that almost all investors highlighted the importance of simplicity, standardization of design, and

The exogenous costs of default are modeled as in Chatterjee and Eyigungor (2012) and they are given by a quadratic loss function for income during a default episode $\phi(y) = max\{d_0 y + d_1 y^2, 0\}$. For $d_0 < 0$ and $d_1 > 0$, the output cost is zero whenever $0 \leq y \leq -\frac{d_0}{d_1}$ and rises more than proportionally with output when $y > -\frac{d_0}{d_1}$. Chatterjee and Eyigungor (2012) show that this type of loss function allows to match reasonably well the sovereign spreads observed in the data. I calibrate (d_0, d_1) in order to match the following statistics for the 2009-2015 period: (i) an average ratio of long-term debt over (annual) GDP of 53%; (ii) the average sovereign spread for Spanish Sovereign long-term bonds (2.3%).[30] On this last point, the sovereign spread in the model is computed as in Chatterjee and Eyigungor (2012). In particular, for the case of non-indexed bonds, I compute an internal rate of return, $r_b(y, b', B', \eta)$ that makes the present discounted value of the promised sequence of future payments (in the absence of default and for a constant endowment process) on a unit bond equal to the unit price. That is, $r_b(.)$ satisfies:[31]

$$q^{ND}(y, b', B', \eta) = \frac{m_b + (1 - m_b)\left[z_b + q^{ND}(y, b', B', \eta')\right]}{1 + r_b(y, b', B', \eta)} \tag{3.4}$$

Similarly, for the GDP-indexed bonds the internal rate of return is defined as the rate $r_B(.)$ that satisfies:

$$p_U^{ND}(y, b', B', \eta) = \frac{m_B + (1 - m_B)\left[z_B(y) + p_U^{ND}(y, b', B', \eta')\right]}{1 + r_B(y, b', B', \eta)} \tag{3.5}$$

The (annualized) sovereign spread for both non-indexed and GDP-linked bonds is

clarity of legal and regulatory treatment (see IMF, 2017 for a description). Other alternatives to this type of bond include "linkers", which are bonds with both principal and coupon linked to the level of a state variable; and "extendibles", which (instead of adjusting the coupon or principal) push out the maturity of a bond if a predefined trigger is breached.

[30]Regarding point (i), given that the paper focuses on long-term debt, instead of including the total stock of central government debt, I only focus on T-Bonds with maturities from 3 to 50 years. For the 2009-2015 period, the ratio of T-Bonds over GDP was around 52.6%; while the total central government debt over GDP was around 65%. In fact, the latter is the value targeted by Hatchondo et al. (2017). Regarding point (ii), the spread was computed using the yields to maturity of a 10-year Spanish and German bond (proxy for the risk-free) for the 2009-2015 period. As a comparison, Hatchondo et al. (2016b) report a sovereign spread of 2.2% for the 8-year Spanish bonds for the 2008-2013 period. The data to compute moments (i) and (ii) comes from the Spanish Tesoro Publico. The data regarding the yields is from the Federal Reserve Bank of St. Louis (FRED).

[31]Notice that if there is no possibility of default, the unit price of a non-indexed bond satisfies: $\bar{q} = \frac{m_b + (1-m_b)[z_b + \bar{q}]}{1 + r_U}$, which in turn implies that: $\bar{q} = \frac{[m_b + (1-m_b) z_b]}{m_b + r_U}$. As $q^{ND}(y, b', B', \eta) \leq \bar{q}$, it follows that $r_b(y, b', B', \eta) \geq r_U$.

then computed as:

$$Spread_i\left(y,b',B',\eta\right)=\left(\frac{1+r_i\left(y,b',B',\eta\right)}{1+r_U}\right)^4-1 \text{ for } i=\{b,B\}$$

I calibrate θ to match the average exclusion period from international markets after a default. I target a value of 3 years, which is the median period of exclusion as reported by Dias and Richmond (2009).[32] Finally, for the recovery rate, I set $f=63\%$ in order to match an average sovereign haircut of 37% as reported by Cruces and Trebesch (2013) for a sample of 180 countries during $1970-2010$.

3.2 Search Frictions Parameters

This subsection describes the calibration of those parameters associated with search frictions in the secondary market for GDP-linked bonds. To start, let $\tilde{m}\left(v,\eta\right)=v^{\chi_1}\eta^{(1-\chi_1)}$ be a constant-returns-to-scale (CRS) Cobb-Douglas matching function. For the quantitative analysis, I assume an increasing-returns-to-scale (IRS) matching function given by $m\left(v,\eta\right)=\left[\tilde{m}\left(v,\eta\right)\right]^{\chi_2}$, where $\chi_2>1$, in order to capture the fact that the probability of finding a dealer (and hence bid-ask spreads and the liquidity premium) depends directly on the size of the secondary market, represented by the measure of constrained sellers η.[33] Appendix E shows the results under a constant-returns-to-scale matching function.[34]

In what follows, I fix the bargaining power $\alpha=0.5$ and the matching function parameter $\chi_1=0.5$ and calibrate the probability of suffering a liquidity shock ζ, the entry cost κ, the discount rate for the constrained investors r_C, and the matching function parameter χ_2, to roughly capture: (i) a bid-ask (BA) spread in the secondary market lower than 20 bps at inception and in the order of 10 bps in the longer-term; (ii) probability of finding a dealer within the quarter that converges to 100% when $\bar{B}=1.0$; (iii) a daily turnover

[32]These authors report the mean and median period of exclusion for both partial and full market re-access after a default. The average length of time it takes for a country to regain partial market access is 5.7 years, while regaining full market access takes 8.4 years on average. Regarding the median period of exclusion, 50% of the countries regain partial market access within 3 years, while it takes 7 years for 50% of the countries to regain full market access. Benjamin and Wright (2009) report similar values for average periods of exclusions.

[33]To ensure that meeting probabilities are between zero and one, I impose: $m\left(v,\eta\right)=Min\left\{\left[\tilde{m}\left(v,\eta\right)\right]^{\chi_2},v,\eta\right\}$.

[34]Under CRS, the probability of finding a dealer does not depend on η and therefore BA spreads and liquidity premia are invariant to the size of the market. Nevertheless, they still depend on the portfolio allocation of (b,B) as this composition affects the default probability, prices, and therefore dealers' incentive to enter the market. Finally, in terms of long-term welfare gains, both cases lead to almost identical results.

Table 3.1: Calibration

Description	Parameter	Value	Targeted Moment / Source
Risk-free rate	r_U	0.01	Standard Values
Discount Factor	β	0.98	Standard Values
Risk Aversion	γ	2	Standard Values
Income Autocorrelation	ρ	0.97	Hatchondo et al. (2016b)
Std of Innovations	σ	0.0104	Hatchondo et al. (2016b)
Income Cost of Defaulting	d_0	-0.7766	2.3% Annual Spread and 53% Debt/GDP Ratio
Income Cost of Defaulting	d_1	0.901	2.3% Annual Spread and 53% Debt/GDP Ratio
Debt Recovery	f	0.62	Cruces and Trebesch (2013)
Probability of Reentering Markets	θ	0.083	Duration of Default - 3 years
Maturity of Non-indexed Debt	m_b	0.0225	Maturity of Bonds - 11 years
Maturity of Indexed Debt	m_B	0.0225	Maturity of Bonds - 11 years
Coupon of Non-indexed Debt	z_b	0.02	Debt Service
Coupon of Indexed Debt	$z_B(y)$	$z_b + 0.1\,(y - \bar{y})$	See text
Bargaining Power	α	0.5	Assumption
Dealers' Operating Cost	κ	0.0004	See text
Discount Rate for $C-types$	r_C	0.012	See text
Prob. of Liquidity Shock	ζ	0.38	See text
Matching Parameter	$\{\chi_1, \chi_2\}$	$\{0.5, 1.4\}$	See text

Notes: This table shows the calibration of the model. Top panel shows the parameters governing the government's problem. Bottom panel shows the parameters governing the search frictions of the secondary market for GDP-linked bonds.

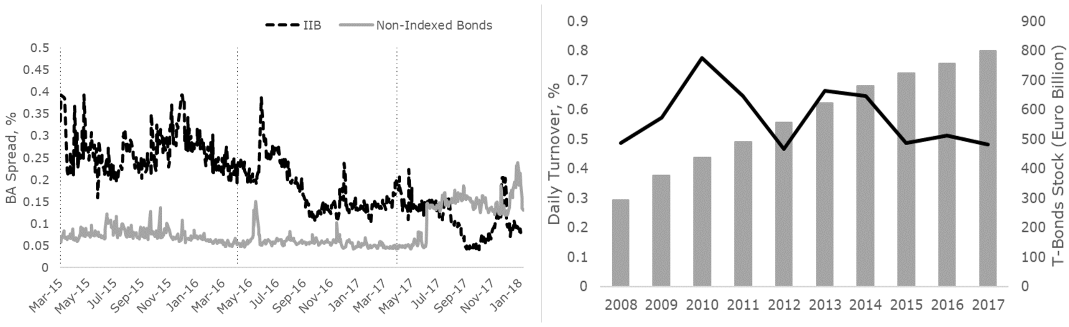
of 0.3% (110% annually) in the secondary market for GDP-linked bonds. The last five rows of Table 3.1 describe the calibrated parameters.

As the model of Section 2 assumes no search frictions for non-indexed bonds, the BA spread targeted in point (i) should be interpreted as the difference between the BA spread for GDP-linked bonds and the BA spread for non-indexed bonds. As GDP-linked bonds have not been issued in big scale by developed countries, providing an estimate of that potential BA spread for the Spanish case is challenging. Using the set of (less developed) countries that have issued GDP-linked bonds in the past (such as Argentina or Greece), presents the issue that spreads in these countries are typically larger than those for developed countries and thus we would be overestimating the potential spread for Spain. Due to these shortcomings, I use as reference the BA spread differential between Spanish inflation-indexed bonds (IIBs) and Spanish nominal (non-indexed) bonds. As the amount outstanding worldwide of IIBs is significantly larger than that of GDP-linked bonds, these values should be considered as a lower bound.

According to data from the Spanish Tesoro Publico, Spain started issuing inflation-indexed bonds after 2014.[35] Using daily data from Bloomberg, I computed the average BA spread across the (five) different IIBs issued since 2015 (left panel of Figure 3.1).

[35] For the 2014-2017 period, issuances of IIBs represented around 7% of total issuances.

Figure 3.1: Spanish Bonds: BA Spreads and Turnover



Notes: The left-panel shows daily BA spreads for Spanish inflation-indexed bonds (IIBs) and Spanish non-indexed bonds for the 2015-2017 period. For IIBs no data is available prior 2014 as Spain did not issue IIBs before that year. For the non-indexed bonds, figures also include those bonds that were issued before 2014 but that had not matured by the end of 2017. Data was retrieved from Bloomberg. Vertical dashed lines indicate days in which Spain issued IIBs. The right panel shows the Spanish stock of stripped and non-stripped government bonds (bars) and daily turnover (line), at a yearly frequency. The depicted turnover measure excludes operations that involve a buy/sell back, repo, or forward transaction. Also, the turnover measure excludes transactions between dealers and between management institutions and their clients. Data comes from the Spanish Tesoro Publico.

Overall, for the 2015-2017 period, the average BA spread was of 19 bps. The figure also reports the average BA spread across nominal (non-indexed) Spanish T-Bonds.[36] For the considered period, the average BA spread for nominal bonds was of 9 bps, implying a 10 bps average difference between the two classes of bonds considered. The difference was even larger for the first two years of the sample and vanished in 2017 due to both an increase in the spreads of the non-indexed bonds and a decrease in the spreads of IIBs. Taking this evidence as a reference point, I target a BA spread differential between GDP-linked bonds and non-indexed bonds of less than 20 bps at inception and in the order of 10 bps in the longer-term, once the stock of GDP-linked bonds has increased.

Regarding point (ii), the model is calibrated such that, for the higher debt limit $\bar{B} = 1.0$, the probability of finding a dealer -$\lambda(v, \eta)$- converges to 100%. As the model described in Section 2 is in discrete time and given that the parameters regarding the government problem (Section 3.1) are calibrated at a quarterly frequency, the minimum trading delay (i.e, $1/\lambda(v, \eta)$) possible is one quarter, which runs counter to the observed delays in any sovereign bond market. In that sense, the model cannot capture trading delays.[37] Nevertheless, the model is still able to match the cost of these trading delays. As shown next, the untargeted measures for liquidity premia are in line with those observed

[36] Sample is limited to availability in the Bloomberg database. Sample includes non-indexed bonds that were issued before and after 2014 and that have not matured by the end of 2017. For both IIBs and non-indexed bonds, the graph excludes outliers in which the average daily BA spread was larger than 40 bps.

[37] A possible solution is to calibrate the model at a weekly or monthly frequency. The problem is that, under the current calibration, they imply a discount factor of $0.98^{1/12} = 0.998$ and $0.98^{1/3} = 0.993$, creating additional difficulties for the convergence of the main algorithm.

for inflation indexed bonds.

Regarding point (iii), the right panel of Figure 3.1 shows yearly data for the daily turnover of Spanish (non-indexed) bonds for the 2008-2017 period. To better capture a measure of turnover that is more closely related to the one of the model presented in Section 2, I exclude operations that involve a buy/sell back, repo, or forward transaction. Also, I exclude transactions between dealers and between management institutions and their clients. Data comes from the Spanish Tesoro Publico. The average for the considered period is around 0.6%.[38] The proposed calibration sets $\zeta = 0.38$ in order to target a daily turnover value for GDP-linked bonds of 0.3% (110% annual turnover), reflecting the fact that indexed bonds are less traded outside the inter-dealer market.[39,40]

4 Quantitative Analysis

I solve the model using value function iteration and linear interpolation over debt levels (b, B), endowment (y), and the measure of constrained agents (η). Appendix C describes the algorithm used to solve the model.

4.1 Prices and Spreads

Before going to the simulations, I start depicting the pricing kernels, the bid-ask spreads, the liquidity premium, and the probability of finding a dealer, for different combinations of the state space and for two different values of $\bar{B}$. Unless otherwise noted, I assume an initial $(B, \eta) = (0, 0)$. Thus, for an issuance of B' units of indexed bonds, the next-period

[38]The Spanish Tesoro Publico does not publish statistics regarding the turnover for inflation-indexed bonds. Using other private sources to retrieve this value (Bloomberg, for instance) has the shortcoming that it is not possible to disentangle whether the transaction was in cash, or if it involved a buy/sell back, repo, or forward transaction. Moreover, we cannot disentangle between transactions performed by two different investors via a broker or transactions performed between dealers in the inter-dealer market.

[39]Notice that quarterly turnover is given by: $\frac{1}{HP(.)}$, where $HP(.)$ is the average holding period given by: $HP(.) = \frac{1}{\zeta} + \frac{1}{\lambda(.)} = 3.632$ under the current calibration. Hence, daily turnover is $\frac{1}{3.632}\frac{1}{90} \approx 0.31\%$.

[40]Claim is based on trading patterns for US TIPS, for which detailed data is available. Fleming and Krishnan (2012) show that the trading patterns for US TIPS are quite different to those observed for nominal US bonds and notes. Consistent with this study, using New York Fed's data of primary dealer transaction volume for the last quarter of 2017, I found that the daily turnover of US TIPS was around one-third of that for nominal bonds and notes (1.53% versus 4.18%, respectively). The difference is even larger if we only consider the inter-dealer market: daily turnover of US TIPS is only one-quarter of that for nominal bonds and notes. However, if we only consider operations between a dealer and another agent (a measure that better tracks the turnover of this model), the daily turnover of US TIPS was around 46% of that for nominal bonds and notes. The current calibration assumes a ratio of 50% for Spain.

measure of constrained investors is $\eta' = \zeta \times B'$, according to equation (2.3).

Figure D.1 (in Appendix D) plots the price for non-indexed bonds as a function of b' (top panels) and for GDP-linked bonds as a function of B' (bottom panels). Notice that both prices are decreasing in (b', B') and increasing in y due to the fact that the probability of default is increasing in the stock of debt and decreasing in the level of endowment. For instance, as the stock of debt converges to zero, notice that q^{ND} converges to 1.2, roughly the price of a risk free bond with maturity $1/m_b$ that pays coupons z_b every quarter. Moreover, notice that the price of GDP-linked bonds reacts only slightly more to a change in y than the price of non-indexed bonds, despite the fact that the payments of the former depends directly on the realization of y. This is explained by the fact that debt is long-term. Under short-term debt, for instance, prices of GDP-indexed bonds would react much more heavily to a change in y.

As in He and Milbradt (2014) and Passadore and Xu (2020), liquidity and default risks are jointly determined. To better understand the role of search frictions in the secondary market for GDP-linked bonds, this section decomposes spreads into a default premium and a liquidity premium. That is, the sovereign spread of a GDP-indexed bond can be decomposed as:

$$Spread_B = Spread_B^{Default} + Spread_B^{Liquidty}$$

In order to quantify the spread associated with the default risk, I first compute the maximum price that an investor with no liquidity problems is willing to pay at issuance, taking as given the equilibrium debt and default policies. To be clear, the policies still take into account the liquidity concerns of the market as a whole, but the pricing is done by an investor that (with certainty) will never suffer from a liquidity shock. That is, in equation (2.10) we set $\zeta = 0$ and obtain:

$$\begin{aligned} p_{NoLiq}^{ND}(y, b', B', \eta) \times (1 + r_U) = \int \{ [1 - h'] \left[m_B + (1 - m_B) \left[z_B(y') + p_U^{ND}(y', b'', B'', \eta') \right] \right] \\ + h' \times \left[p_U^D(y', b', B', \eta') \right] \} dF(y' \mid y) \end{aligned} \tag{4.1}$$

Using this price, I then compute the internal rate of return for this investor and its annualized spread ($Spread_B^{Default}$). The liquidity premium is given by the differential between $Spread_B$ and $Spread_B^{Default}$. It is easy to show that this liquidity premium is increasing in ζ and decreasing in $\lambda(v, \eta)$.

For the $\bar{B} = 0.4$ case , Figure 4.1 plots the liquidity premium for GDP-linked bonds (left panel), together with the BA spreads in the secondary market (right panel).[41] The initial portfolio is given by $(b, B) = (X, 0)$ and, in all cases, the next-period total stock of debt (i.e., $b' + B'$) is held constant at X.[42] By endogenizing the measure of dealers, both the liquidity premium and the BA spreads depend on the amount of bonds outstanding B' (as this measure impacts η' directly). Overall, both BA spreads and liquidity premia are decreasing in the stock of GDP-indexed bonds. For example, BA spreads decrease from around 20 bps when the stock of GDP-indexed bonds is almost zero, to less than 12 bps as B' increases. Similarly, the liquidity premium decreases from 60 bps to less than 35 bps.[43]

Even when BA spreads and the liquidity premium are decreasing with market size, they do not converge to zero. This is because the entry cost of dealers, κ, is assumed to be constant and independent of market size. Therefore, even if there are many constrained sellers and dealers in the market, the bid price must be lower than the price at issuance, so that dealers can pay the fixed cost κ.

Finally, BA spreads and liquidity premia are increasing in the total stock of debt (X) and they display a highly countercyclical behavior.[44] Why is this the case? An increase in X/y can be viewed as an increase in the probability of default. If the country defaults, then prices of both indexed and non-indexed debt plummet. Given the smaller magnitude of prices, the difference between p_U^{ND} and p_C^{ND} also decreases. As dealers' profit depends positively on this difference, fewer dealers enter the market (for any given η), implying a lower probability of finding a dealer as shown in Figure D.2 (in Appendix D). Finally, because of the Nash bargaining assumption, a lower probability of finding

[41] Appendix Figure D.3 shows the case for $\bar{B} = 1.0$. Given that the ask price in the secondary market is given by p_U^{ND}, the bid-ask spread is computed as:

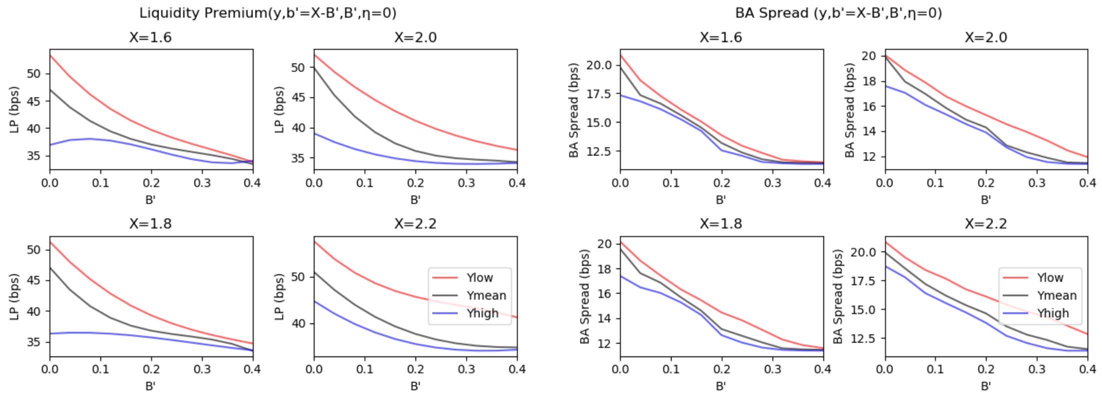
$$\left(\frac{p_U^{ND} - p_B^{ND}}{0.5\left(p_U^{ND} + p_B^{ND}\right)} \right) = \frac{2\,(1-\alpha)\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]}{p_U^{ND} + p_B^{ND}}$$

[42] Therefore an issuance of GDP-linked bonds implies a buy-back of the same size of non-indexed bonds.

[43] In Passadore and Xu (2020), as the probability of finding a dealer is constant and independent of the size of the market, the liquidity component of the spread is almost constant during good times and only increases when the sovereign is close to default (because they exogenously assume that dealers' bargaining power is higher when the sovereign is in default).

[44] The latter is consistent with Andreasen et al. (2017), who show that US TIPS liquidity premia are characterized by a clear countercyclical variation, given their positive relationship with the VIX for the S&P 500. They argue that this may be explained by the fact that higher uncertainty tends to increase the risk associated with the future resale price of any security and therefore investors and intermediaries require a higher premium for holding less liquid assets.

Figure 4.1: BA Spreads and Liquidity Premium ($\bar{B} = 0.4$)



Notes: Liquidity premium (left panel) and bid-ask spreads (right panel) for GDP-linked bonds for different combinations of the state space. The initial portfolio is given by $(b, B) = (X, 0)$ and, in all cases, the next-period total stock of debt (i.e., $b' + B'$) is held constant at X. $y_{Low} = \bar{y} - \sigma_y$ and $y_{High} = \bar{y} + \sigma^y$, where σ^y is computed after simulating the economy 10,000 times and dropping those paths were the sovereign government defaulted (see section 4.2 for additional details). The liquidity premium was smoothed using a third order degree polynomial to avoid some small kinks. Figures correspond to the debt limit of $\bar{B} = 0.4$ (10% of annual GDP).

a dealer translates into larger BA spreads in the secondary market. Investors anticipate this larger spread and will require a premium for the higher illiquidity of the bond.

4.2 Simulations

I first solve a simpler version of the model presented in Section 2 by considering a scenario in which the government cannot issue GDP-linked bonds ($\bar{B} = 0$). Then, following the methodology of Hatchondo et al. (2017), I measure the effects of an unanticipated announcement stating that the government, from that time onwards, will be able to issue GDP-linked debt ($\bar{B} > 0$). I perform the same analysis under two cases: (i) no-search frictions in the secondary market for GDP-indexed bonds (I capture this in the baseline model by setting $\zeta = 0$); (ii) search frictions in the market of GDP-linked bonds. I repeat the analysis for two different values of $\bar{B}$ and describe government's optimal portfolio allocation and welfare gains from issuing GDP-linked bonds. Table 4.1 summarizes the three different scenarios.

Table 4.1: Scenarios

Description	Scenario 1	Scenario2	Scenario 3
Indexed Bonds?	No	Yes	Yes
Search Frictions?	-	No	Yes

Table 4.2: Targeted and Untargeted Moments

Moment	No GDP bonds	No-Search Frictions		Search Frictions	
	$\bar{B}$=0	$\bar{B} = 0.4$	$\bar{B} = 1.0$	$\bar{B} = 0.4$	$\bar{B} = 1.0$
	(i)	(ii)	(iii)	(iv)	(v)
Defaults in 100 Years	5.10	4.78	4.37	5.01	4.43
Debt / Quarterly-GDP	2.08	2.07	2.05	2.09	2.08
Non-indexed Debt / Quarterly-GDP	2.08	1.71	1.10	1.72	1.17
Indexed Debt / Quarterly-GDP	-	0.36	0.96	0.36	0.91
Annual Spread for Non-indexed Debt	2.34%	2.28%	2.25%	2.29%	2.26%
Spread Volatility for Non-indexed Debt	1.04%	1.03%	1.00%	1.03%	1.00%
Annual Spread for Indexed Debt	-	2.59%	2.54%	2.99%	2.91%
Spread Volatility for Indexed Debt	-	0.63%	0.59%	0.67%	0.62%
$std(c)/std(y)$	1.14	1.11	1.05	1.12	1.07
$corr(TB/y, y)$	-0.61	-0.65	-0.68	-0.65	-0.69
Secondary Market for SCDIs					
Annual Turnover	-	-	-	108.57%	110.82%
Liquidity Premium (Bps)	-	-	-	36.10	33.58
BA spread (Bps)	-	-	-	12.08	11.10

Notes: The table shows the targeted and untargeted moments based on 10,000 simulations. Only simulations in which the government does not default in the last 100 quarters are considered. Table reports averages across the last 80 quarters.

4.2.1 Economy without GDP-indexed Bonds

The first column of Table 4.2 reports moments based on simulations for the economy without GDP-indexed bonds ($\bar{B} = 0$). Moments are computed for sample paths without defaults. In particular, I generate $10,000$ sample paths of 300 periods (quarters) each. Given that default episodes are not the topic under analysis, out of these $10,000$ samples I only keep those in which the government does not default in the last 100 periods (except for the first row of the table). I compute moments based on the last 80 periods (20 years) of the non-discarded samples.[45]

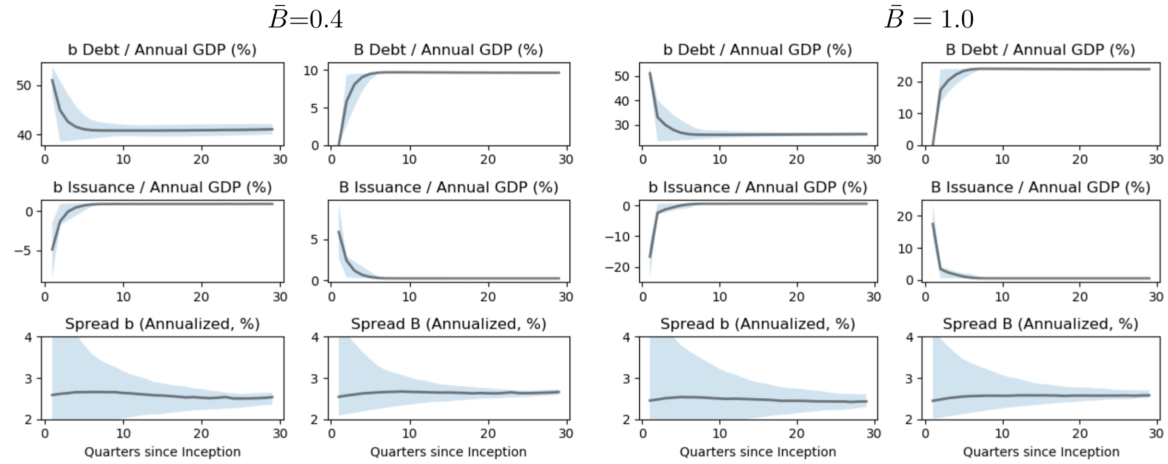
Column 1 of Table 4.2 shows that the model with no GDP-linked bonds captures well all the targeted moments: debt over GDP ratio and annual sovereign spread. Also, the model is fairly consistent with non-targeted moments. For example, model simulations show an average of 5 default episodes per 100 years, which is in the range of the historical number of default episodes for Spain reported by Reinhart et al. (2003).[46] Also, consumption is around 14% more volatile than output and the trade balance is highly counter-cyclical, consistent with empirical evidence for Spain.[47]

[45] This methodology is similar to the one employed by Hatchondo et al. (2016b).

[46] Using historical data with default episodes, Reinhart et al. (2003) indicate that Spain defaulted on its external debt 13 times between 1500 and 1900, with the first default recorded in 1557 and the last in 1882. However, 7 of these default episodes were observed during 1801-1900.

[47] Hatchondo et al. (2016b) report that unlike the stylized fact for many developed countries, consumption was 15% more volatile than output for the Spanish economy during the 1995-2013 period. For the

Figure 4.2: Introducing GDP-linked bonds under No-Search Frictions



Notes: Figure shows debt levels (as a share of annual GDP), quarterly issuances (as a share of annual GDP), and annualized sovereign spreads after the introduction of GDP-indexed bonds. The black solid line represents the average path across 10,000 simulations starting at the mean values for endowment and non-indexed debt (computed through simulations). The shaded blue area corresponds to the average path across simulations for different initial values of y and b. In particular, $y_0 = \bar{y} \pm \sigma^y$ and $b_0 = \bar{b} \pm \sigma^b$, where both standard deviations are computed through simulations. No search frictions are assumed in the secondary market for GDP-linked debt. Left panel corresponds to the debt limit of $\bar{B} = 0.4$ (10% of annual GDP). Right panel shows results for a debt limit of $\bar{B} = 1.0$ (25% of annual GDP). Only paths without defaults are considered.

4.2.2 Economy with GDP-indexed Bonds and No-Search Frictions

Figure 4.2 shows the effects of introducing GDP-linked bonds under the assumption of no-search frictions in its secondary market. The black solid line represents the average path across 10,000 simulations starting at the mean values for endowment and non-indexed debt: $\left(\bar{y}, \bar{b}\right)$. The shaded blue area corresponds to the average path across simulations for different initial values of y and b, involving economic expansions (recessions) with low (high) values of non-indexed debt. For the two considered debt limits ($\bar{B} = 0.4$ and $\bar{B} = 1.0$), and independently of the initial state considered, immediately after the announcement, the government issues GDP-linked debt up to (almost) the specified limit. On the other hand, the stock of non-indexed debt decreases sharply, as the government is heavily buying back this type of debt. The adjustment period is small (particularly when $\bar{B} = 1.0$), as almost all changes are observed within 3-4 quarters after the introduction of GDP-linked debt.

Columns 2 and 3 of Table 4.2 show that, in the long-term, the average stock of GDP-linked debt is almost at its limit ($\bar{B}$). The stock of non-indexed debt, on the other hand, decreases with respect to the baseline model, while the total stock of debt is fairly constant. Finally, the spreads of non-indexed bonds are (almost) unaffected by

same sample period, they report a negative correlation between the trade balance and GDP of −72%. Both results are in line with the results of this paper.

the introduction of GDP-linked debt. For the higher debt limit of $\bar{B} = 1.0$, for example, the reduction in the spreads of non-indexed debt is around 10 bps only, consistent with the fact that the frequency of default is roughly the same across the different scenarios, as shown in Table 4.2. The latter contradicts previous claims made in the literature that the introduction of GDP-indexed debt can help reduce the likelihood of debt crises (see Borensztein and Mauro, 2004, for example), but is in line with previous quantitative models featuring GDP-linked bonds (Onder 2017, for example). To conclude, notice that the volatility of the spreads for GDP-indexed bonds is considerably smaller than the volatility of the spreads for non-indexed bonds. As we are under the no-search frictions assumption, prices are independent of η. Hence, from equations 3.4 and 3.5, the spreads are given by:

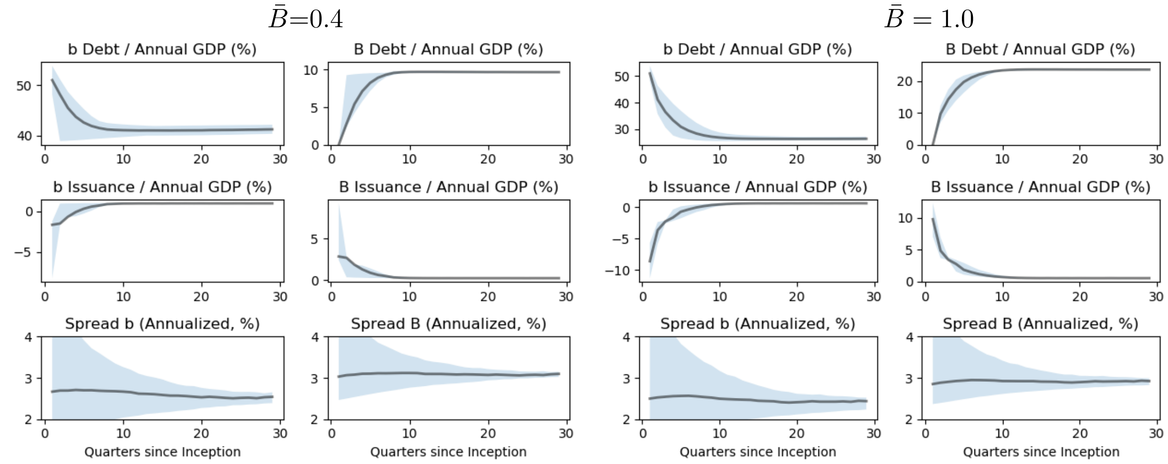
$$
\begin{aligned}
r_b\left(y, b', B'\right) &= \frac{m_b + \left(1 - m_b\right) z_b}{q^{ND}(y, b', B')} - m_b \\
r_B\left(y, b', B'\right) &= \frac{m_B + \left(1 - m_B\right) z_B\left(y\right)}{p_U^{ND}(y, b', B')} - m_B
\end{aligned}
$$

In an economic downturn that increases the probability of default, both prices $q^{ND}(.)$ and $p_U^{ND}(.)$ decrease. However, the coupon payments also decrease in the case of GDP-linked bonds, implying that the increase in the spreads of GDP-indexed bonds is smaller than the increase in the spreads of non-indexed debt. An analogous analysis can be done for an economic expansion, and we obtain that the volatility of GDP-linked bonds spreads is smaller than the volatility of the spreads of non-indexed bonds. The previous "non-linearity" is also behind the result that the average spread of GDP-linked bonds is slightly higher than the spreads of non-indexed bonds.

4.2.3 Economy with GDP-indexed Bonds and Search Frictions

Figure 4.3 shows the effects of introducing GDP-linked bonds under the assumption of search frictions in its secondary market. Overall, GDP-linked bonds are introduced at a slightly slower pace under search frictions (with respect to the no-frictions case), particularly when $\bar{B} = 1.0$. In other words, the government does not find optimal to issue GDP-linked bonds all the way to the limit ($\bar{B}$) at once, but prefers to spread the issuances across time. In Section 4.4, I provide an explanation to this behavior. Finally, due to the liquidity premium, the spreads of indexed bonds are $30-40$ bps higher, relative to the no-frictions case.

Figure 4.3: Introducing GDP-linked bonds under Search Frictions



Notes: Figure shows debt levels (as a share of annual GDP), quarterly issuances (as a share of annual GDP), and annualized sovereign spreads after the introduction of GDP-indexed bonds. The black solid line represents the average path across 10,000 simulations starting at the mean values for endowment and non-indexed debt. The shaded blue area corresponds to the average path across simulations for different initial values of y and b. In particular, $y_0 = \bar{y} \pm \sigma^y$ and $b_0 = \bar{b} \pm \sigma^b$, where both standard deviations are computed through simulations. Search frictions are considered in the secondary market for GDP-linked debt, as explained in Section 2. Left panel corresponds to the debt limit of $\bar{B} = 0.4$ (10% of annual GDP). Right panel shows results for a debt limit of $\bar{B} = 1.0$ (25% of annual GDP). Only paths without defaults are considered.

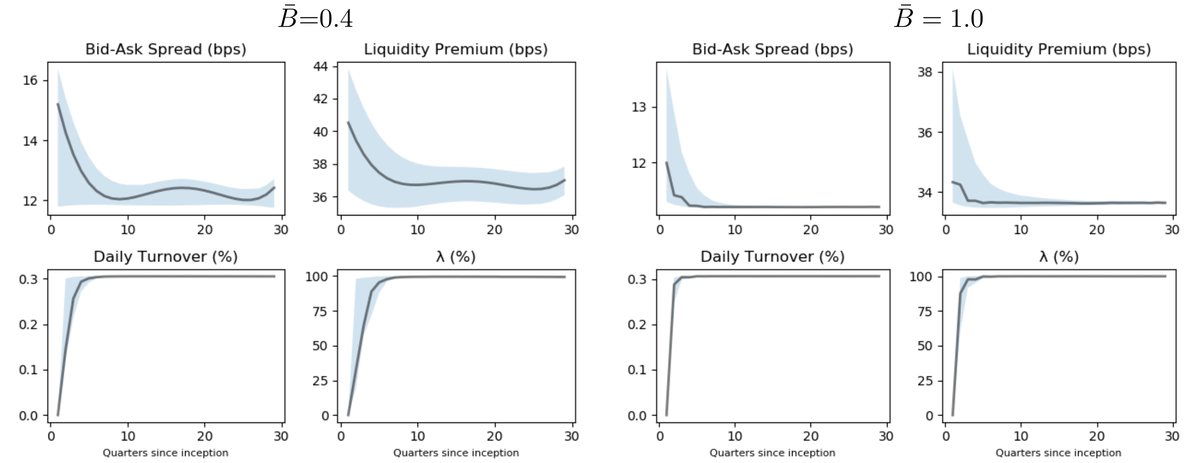
Figure 4.4 provides different measures to describe the search frictions in the secondary market for GDP-indexed debt at the time of inception of such bonds. First, notice that both BA spreads and the liquidity premium are smaller when the debt limit $\bar{B}$ is larger. Second, the figure shows that both BA spreads and the liquidity premium are decreasing in time as the stock of indexed-debt increases. In the long-term, the liquidity premium converges to around 34 bps while the BA spread converges to 11 bps (as shown in Table 4.2). This last result is consistent with bid-ask spreads observed for Spanish inflation indexed bonds.

Third, the figure shows a daily turnover ratio of around 0.3% in the long-term, but this value is significantly smaller at the introduction of these bonds as the number of dealers in the market is small.[48] Finally, the figure shows that the probability for a constrained seller to find a dealer within the quarter -$\lambda(v, \eta)$- increases rapidly in time. For the upper debt limit case, the probability converges quickly to 100%, while it converges to around 90% for the lower debt limit. At inception, however, this probability can be significantly lower. For instance, five quarters after the introduction of these bonds the probability of

[48]Turnover is computed as the inverse of the holding period (HP), defined as: $HP(v, \eta) = \frac{1}{\zeta} + \frac{1}{\lambda(v,\eta)}$. A similar picture emerges if turnover is computed as volume traded as a share of the amount outstanding. In the latter case, to avoid double counting of transactions, only those transactions involving a dealer and a constrained seller should be considered.

finding a dealer is below 80% when $\bar{B} = 0.4$.

Figure 4.4: Introducing GDP-linked bonds under Search Frictions



Notes: Figure shows different measures regarding the search frictions in the secondary market for GDP-indexed bonds, under different initial conditions for initial endowment and initial stock of non-index debt. The black solid line represents the average path across $10,000$ simulations starting at the mean values for endowment and non-indexed debt. The shaded blue area corresponds to the average path across simulations for different initial values of y and b. In particular, $y_0 = \bar{y} \pm \sigma^y$ and $b_0 = \bar{b} \pm \sigma^b$, where both standard deviations are computed through simulations. Left panel corresponds to the debt limit of $\bar{B} = 0.4$ (10% of annual GDP). Right panel shows results for a debt limit of $\bar{B} = 1.0$ (25% of annual GDP). Only paths without defaults are considered.

Although there aren't studies that estimate the liquidity premium component of the (few) GDP-linked bonds that have been issued in the past (neither for Spanish inflation indexed bonds), there are many studies that compute the liquidity premium for inflation-indexed bonds in the US and UK. As reviewed by Blanchard et al. (2016), the consensus regarding the US TIPS is that the liquidity premium has been decreasing as the stock and transaction volumes of these bonds have increased. For instance, upon the TIPS introduction in January 1997, the liquidity premium was around 100 bps according to D'Amico et al. (2014) and gradually declined during the first-half of the 2000s. Pflueger and Viceira (2013) show that TIPS trading volume (relative to nominal bonds) has a negative and significant relationship on TIPS yields differential, indicating that search frictions impacted inflation-indexed bond prices during the early period of TIPS issuance. In particular, they show that the observed increase in TIPS relative trading volume from 1999 to 2004 was associated with a decrease in the TIPS liquidity premium of 48 bps.[49] Results in Figure 4.4 are also consistent with these trends.

[49] Pflueger and Viceira (2013), for a sample period covering 1999-2010, estimate an average liquidity premium for inflation-indexed bonds of 69 bps for the US TIPS and 50 bps for the UK. Auckenthaler et al. (2015) estimate an average liquidity premium of 56 bps for the case of the US TIPS, 118 bps for the UK, and 154 bps for Canada. Hördahl and Tristani (2014) and D'Amico et al. (2014) present similar estimates and trends for the US.

4.2.4 Liquidity Premium During a Crisis

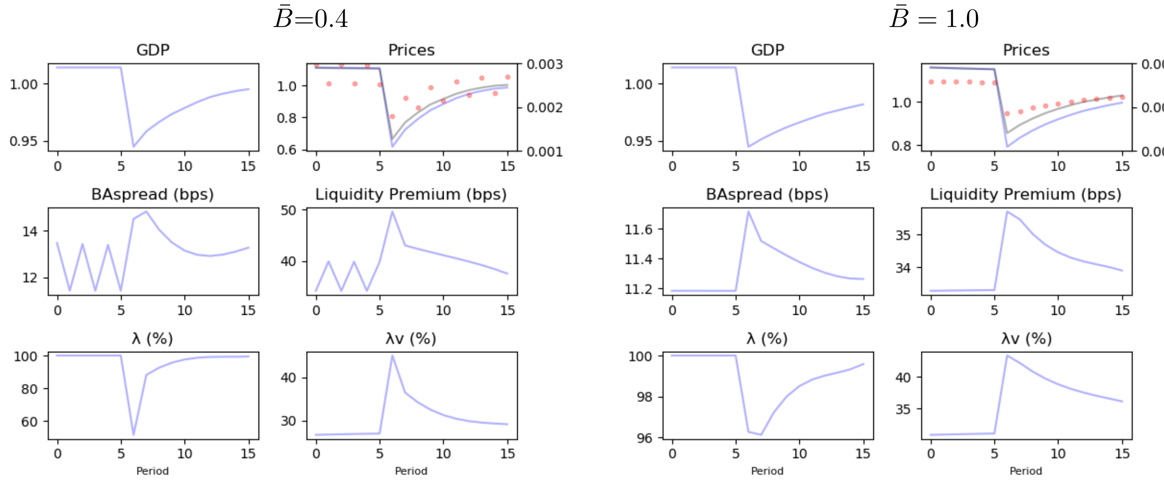
A salient feature of the US TIPS during the last global crisis was a large increase in their liquidity premium during 2008-2009: up to 300 bps according to D'Amico et al., 2014 or 150 bps according to Pflueger and Viceira, 2013. After the crisis, the premium had largely returned to their pre-crisis levels (D'Amico et al., 2014). The large spike in TIPS liquidity premium during the crisis was accompanied by a (narrower) increase in BA spreads and a sharp decrease in transaction volumes. Haubrich et al. (2006) and D'Amico et al. (2014) show that the BA spread on the ten-year TIPS increased from 1 bps to over 10 bps during the crisis, before settling down to around 4 bps after 2010. Moreover, weekly transaction volumes decreased from 10 billion at the beginning of 2008 to 5 billion in 2009-2010.

In this subsection, I show that even with risk-neutral investors, the model is somewhat able to replicate an increase in liquidity premia during a crisis. In particular, I study how a (domestic) crisis affects BA spreads and liquidity premia of GDP-indexed bonds. I artificially generate a crisis by exogenously (and unexpectedly) decreasing current GDP.

Figure 4.5 plots the effects of a domestic crisis on prices, BA spreads, liquidity premium, and on the probability of finding a dealer to trade with. Simulations start on steady state values. I generate a temporary recession by exogenously decreasing current endowment by two standard deviations (around 6%). The economy then recovers following its Markov process. Only simulations in which the sovereign government does not default are considered.

First, notice that prices for GDP-linked bonds p_U^{ND} (blue solid line) and for nominal bonds q^{ND} (black solid line) decline sharply. Importantly, notice that the magnitude of these declines is similar, implying that the price of GDP-indexed bonds declines mostly because of the higher probability of default and not because of the lower coupon payment -$z_B(y)$-. Given the smaller magnitude of prices, the difference between p_U^{ND} and p_C^{ND} (red dotted line) also decreases. As dealers' profit depends positively on this difference, fewer dealers enter the market, implying a lower probability of finding a dealer. The decrease in this probability results in a lower (effective) bargaining power of constrained sellers and therefore the bid price at which they can sell their position decreases, implying an increase in bid-ask spreads. This increase leads investors to demand a higher compensation at issuance for buying GDP-linked bonds (i.e, a higher liquidity premium).

Figure 4.5: Effects of a Crisis on BA Spreads and Liquidity Premium



Notes: Figure shows the effects of an economic downturn (a decrease of 2 standard deviations in the endowment) in prices, BA spreads, and liquidity premium of GDP-linked bonds. The blue solid line represents the average path across 10,000 simulations starting at the mean values for endowment and non-indexed debt. Search frictions are considered in the secondary market for GDP-linked debt, as explained in Section 2. Left panel corresponds to the debt limit of $\bar{B} = 0.4$ (10% of annual GDP). Right panel shows results for a debt limit of $\bar{B} = 1.0$ (25% of annual GDP). Only paths without defaults are considered.

Notice that the effect on the liquidity premium depends on the amount outstanding of GDP-linked bonds. For the lower debt limit of $\bar{B} = 0.4$, the left panel of Figure 4.5 shows a sharp increase of around 35% in the liquidity premium. In particular, BA spreads increase from 12 to 14 bps, while the liquidity premium increases from 35 to 47 bps. For the upper debt limit of $\bar{B} = 1.0$, the increases in both BA spreads and liquidity premium are significantly smaller. This is explained by the fact that $\lambda(v, \eta)$ remains almost constant in this case, implying that the (effective) bargaining power of constrained sellers remained almost constant.

Summing up, even with risk-neutral investors, the model is able to capture a sizable increase in both BA spreads and liquidity premia during a crisis. The magnitudes, however, are smaller than those observed for the US TIPS in the last global recession. To better capture this last fact, it may be necessary to introduce some sort of risk aversion for foreign investors and further work is needed.

4.3 Welfare Gains

This section computes welfare gains derived from the introduction of GDP-indexed bonds under no-search frictions and under search frictions. I measure welfare gains in terms of certainty equivalent consumption. That is, for every initial state (y, b), I compute by how much we would need to change consumption of the representative household for it to be indifferent between living in the economy with no GDP-indexed bonds versus living in an economy with GDP-indexed bonds. Formally, let $(c^{\star}, V^{\star})$ and $\left(\tilde{c}, \tilde{V}\right)$ denote optimal consumption and the value function in the economy with no GDP-linked debt and in the economy with GDP-linked debt, respectively. The certainty equivalent consumption, $\omega\,(y, b, 0)$, is given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(\left[1+\omega\,(y,b,0)\right] c_t^{\star}\right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(\tilde{c}_t\right)$$

Exploiting the homogeneity of the power-utility function, the previous equation can be written as:

$$\omega\,(y,b,0) = \left[\frac{\tilde{V}\,(y,b,0)}{V^{\star}\,(y,b)}\right]^{\frac{1}{1-\gamma}} - 1 \tag{4.2}$$

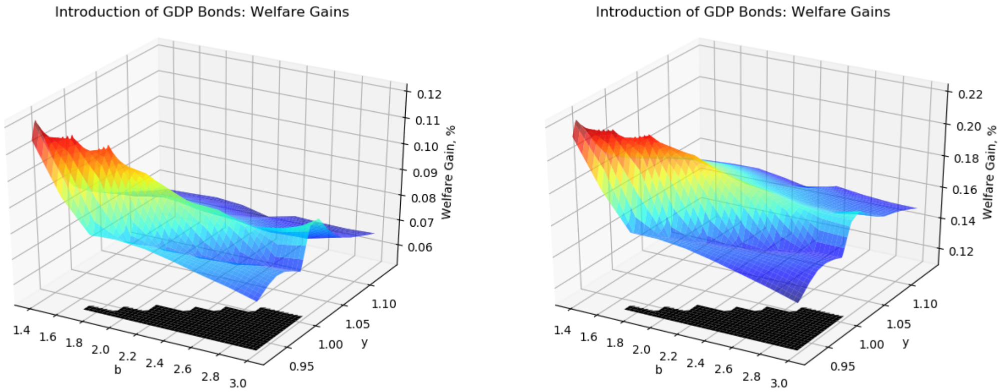
The top panel of Figure 4.6 shows the ex-ante welfare gains derived from the introduction of GDP-indexed bonds under the no-search frictions case. The gains are in the order of 0.07 to 0.22 percent of certainty equivalent consumption (depending on the initial state and on the announced debt limit). These welfare gains arise from a higher degree of risk sharing between the risk averse sovereign government and the risk neutral foreign investors that leads to a decrease in the volatility of consumption relative to output (as shown in Table 4.2). The reported welfare gains are slightly lower than what previous studies have found for the Argentine case (Sandleris et al., 2011 and Hatchondo and Martinez, 2012).

The black regions in Figure 4.6 denote those points of the state space in which the sovereign decides to default. Notice that, as we move towards this area, welfare gains derived from the introduction of GDP-linked bonds decrease. For these points of the state-space, introducing GDP-linked bonds is not highly valued by the representative household because it cannot prevent a default crisis. Out of the default region, notice that the largest welfare gains are obtained when the endowment is smaller. The risk-sharing benefits of GDP-indexed debt are larger when y is low, as this type of debt allows smaller interest payments until the economy recovers. Of course, when the endowment is below its mean, the price of GDP-linked bonds is lower than the price of non-indexed bonds. However, this extra cost of issuing GDP-linked bonds is more than compensated by the lower interest payments that the sovereign will make until the economy recovers. This follows from the fact that the sovereign is risk averse, investors are risk neutral, and

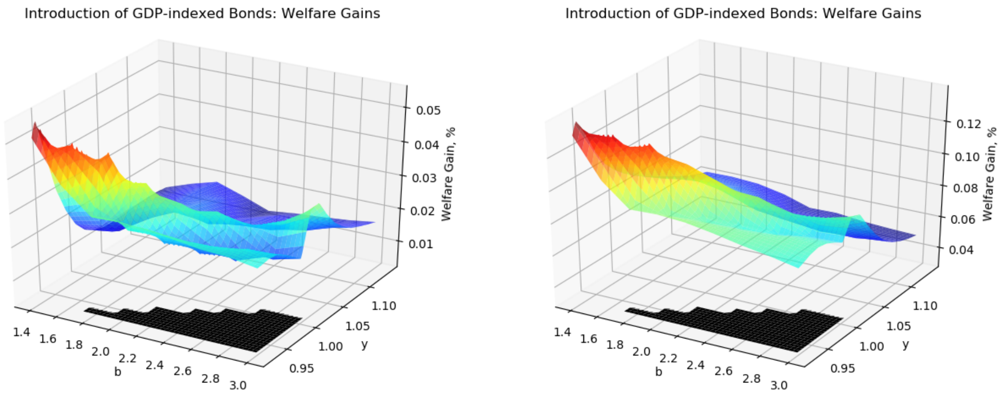
Figure 4.6: Welfare Gains and Default Regions

$\bar{B} = 0.4$ $\bar{B} = 1.0$

No-Search Frictions: $\omega\,(y, b, 0)$



Search Frictions: $\omega^{SF}\,(y, b, 0, 0)$



Notes: Figure shows welfare gains derived from the introduction of GDP-linked bonds under the assumption of no-search frictions (top panel) and under the assumption of search frictions (bottom panel) in the secondary market for GDP-linked debt, for different initial values of endowment and debt (y, b). Dark area denotes points of the state space in which the sovereign defaults.

the time it will take until the economy recovers is stochastic.

The bottom panel of Figure 4.6 shows that, under search frictions, welfare gains of issuing GDP-indexed bonds are cut in half: they are in the order of 0.02 to 0.12 percent of certainty equivalent consumption, depending on the initial state for y and b and on the announced debt limit for GDP-linked bonds. Table 4.3 shows that the decrease in welfare gains is proportionally larger for the lower debt limit ($\bar{B} = 0.4$), as this case is associated with a slightly larger liquidity premium due to the smaller size of the secondary market. Overall, the small welfare gains that we obtained once search frictions are considered may help to explain why sovereign countries have not issued GDP-indexed bonds.

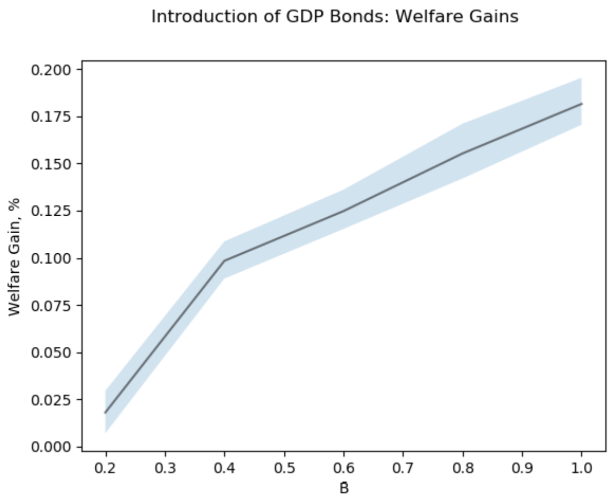
Interestingly, in both scenarios, notice that welfare gains for the case of $\bar{B} = 1.0$ less than double welfare gains for the case of $\bar{B} = 0.4$, indicating diminishing marginal welfare gains on GDP-linked bond issuances. To see this last point clearer, Figure 4.7

Table 4.3: Welfare Gains Ratios

	$\bar{B} = 0.4$		$\bar{B} = 1.0$	
	No-Search Frictions	Search Frictions	No-Search Frictions	Search Frictions
Welfare Gains, %	0.0913	0.0280	0.1912	0.1003
Ratio, %	-	30%	-	52%

Notes: Comparison of welfare gains under search frictions and under the case with no frictions in the secondary market for GDP-linked bonds. Results correspond to the case in which $(y_0, b_0) = (\bar{y}, \bar{b})$. That is, the initial values are the unconditional mean of the endowment process and the average stock of non-indexed bonds (before GDP-linked bonds are introduced).

Figure 4.7: Diminishing Marginal Welfare Gains (No-search Frictions)



Notes: Figure shows welfare gains for different values of $\bar{B}$ under the no-search frictions scenario. Black solid line indicates the case in which the economy initial state is $(\bar{y}, \bar{b}, 0)$. The shaded blue area encompasses other initial states. Model is solved for $\bar{B} = \{0.2, 0.4, 0.6, 0.8, 1.0\}$. Linear interpolation is used for other debt limits.

plots welfare gains for different values of $\bar{B}$, under the no-search frictions scenario only. Notice that welfare gains increase rapidly for $\bar{B} < 0.4$ but from then on they increase at a slower pace.

To conclude, as our (untargeted) estimate of the liquidity premium is towards the lower end of previous studies estimating the liquidity premium for inflation-linked bonds, the welfare gains computed in this section should be interpreted as a "most favorable case". A higher target for the liquidity premium would decrease these welfare gains even further.

4.4 Why are GDP-indexed bonds introduced differently under search frictions?

As already described in Figure 4.3, GDP-linked bonds are introduced at a slightly slower pace under search frictions (with respect to the no-frictions case), particularly when $\bar{B} = 1.0$. In this subsection, I analyze the forces behind this difference, highlighting the trade-off that the government faces when choosing its optimal portfolio allocation.

Following the methodology in Bianchi et al. (2018), I analyze different portfolios that deliver the same level of current consumption. Along the section, I assume that the bond prices and the value function are differentiable.[50] To simplify the analysis, I further assume that $f = 0$ so there is no recovery value. Consider a consumption target $\bar{c}$. Let $X = \{y, b, B, \eta, \bar{c}\}$ denote the vector of initial states and the consumption target. Using the resource constraint, all the possible combinations of (b', B') that deliver a level of consumption equal to $\bar{c}$ are given by:

$$\begin{aligned} H\left(b', B'; X\right) = y - \bar{c} - b\left[(1-m_b)\, z_b + m_b)\right] - B\left[(1-m_B)\, z_B\left(y\right) + m_B)\right] \\ + p_U^{ND'}\left[B' - (1-m_B)\, B\right] + q^{ND'}\left[b' - (1-m_b) b\right] = 0 \end{aligned} \tag{4.3}$$

where $p_U^{ND'} \equiv p_U^{ND}\left(y, b', B', \eta\right)$ and $q^{ND'} \equiv q^{ND}\left(y, b', B', \eta\right)$. Suppose that the government issues additional GDP-linked bonds and buys back non-indexed bonds with the proceeds. How many non-indexed bonds can the government buy back? Define $\tilde{b}\left(B', X\right)$ as the stock of non-indexed bonds that solves 4.3 for each value of B', given X. In other words, $\tilde{b}\left(B', X\right)$ is the amount of non-indexed bonds after the buy-back, for a given initial value of X and an issuance of $\left[B' - (1-m_B)\, B\right]$. Applying the implicit function theorem to 4.3, the rate at which the government can buy back non-indexed bonds is given by:

$$\frac{-d\tilde{b}\left(B', X\right)}{dB'} = \frac{p_U^{ND'} + \frac{\partial p_U^{ND'}}{\partial B'} \times \left[B' - (1-m_B)\, B\right] + \frac{\partial q^{ND'}}{\partial B'}\left[\tilde{b}\left(B', X\right) - (1-m_b) b\right]}{q^{ND'} + \frac{\partial p_U^{ND'}}{\partial b'} \times \left[B' - (1-m_B)\, B\right] + \frac{\partial q^{ND'}}{\partial b'} \times \left[\tilde{b}\left(B', X\right) - (1-m_b) b\right]} \tag{4.4}$$

This expression describes the rate at which non-indexed debt can be replaced with indexed-debt. By issuing one additional unit of B', the government gets $p_U^{ND'}$ units of consumption that can be allocated to buy back non-indexed debt (whose price is

[50] For the canonical model of default, Clausen and Strub (2017) show that the government's objective function is continuously differentiable at the optimal choices and a particular version of the envelope theorem applies. Although this may not be exactly the case for the current model, I simply assume differentiability to highlight the trade-offs of the government. It is important to remember that the algorithm used to solve the model does not rely on (global) differentiability.

$q^{ND'}$). Moreover, the government takes into consideration that by issuing one more unit of indexed debt it also affects the price of both types of debt due to a change in the probability of default (an a change in the next-period liquidity premium). The lower the impact of an additional unit of B' on $p_U^{ND'}$ and $q^{ND'}$ (i.e., the larger $\frac{\partial p_U^{ND'}}{\partial B'}$ and $\frac{\partial q^{ND'}}{\partial B'}$) and the higher the impact of a decrease in b' on both prices (i.e., the lower $\frac{\partial p_U^{ND'}}{\partial b'}$ and $\frac{\partial q^{ND'}}{\partial b'}$), the larger the amount of bonds that the government can buy back for a given B' and X.[51].

Assuming an initial state given by $(y, b, B, \eta) = \left(\bar{y}, \bar{b}, 0, 0\right)$, Figure 4.8 plots the percentage change in the total stock of debt if the government decides to issue dB' units of GDP-linked bonds (i.e., $100 \times \frac{d\tilde{b}(B',X)+dB'}{\bar{b}}$).[52] Notice that under no-search frictions (red line), the government can decrease its total stock of debt by issuing GDP-linked bonds and buying back non-indexed debt with the proceeds. In other words, there are no net-costs from issuing GDP-linked bonds. As, in general, the government prefers a lower stock of debt and a larger share of GDP-linked bonds in its portfolio, it has a strong incentive to issue at once the maximum amount possible of GDP-linked bonds (as shown in Figure 4.2).

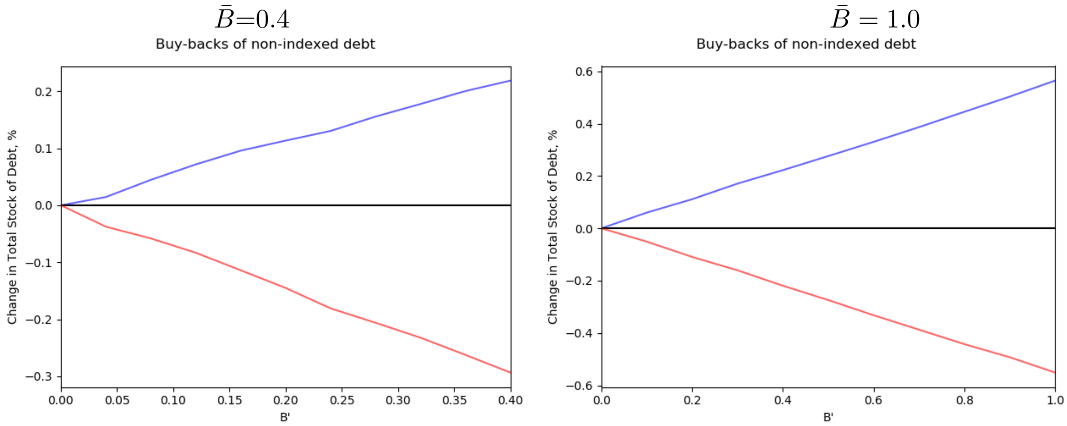
On the other hand, due to the liquidity premium, the government can buy back fewer units of non-indexed bonds for any given issuance B'. In particular, Figure 4.8 shows that the total stock of debt increases if the government issues GDP-linked bonds and buys back non-indexed debt with the proceeds (blue line). Under this scenario, therefore, there is an interesting trade-off between the benefits (i.e., a better consumption smoothing) from issuing GDP-linked bonds and its associated costs. As shown in Figure 4.3, the government optimally chooses to spread the issuances of GDP-linked bonds across time. I next discuss the forces behind this decision.

How does the government decide among all the different combinations of GDP-linked bonds and non-linked debt? Given that current utility is fixed by current consumption $\bar{c}$, the optimal portfolio needs to maximize the expected continuation value. Assuming that the government optimally chooses consumption and portfolios from next period onward,

[51] Quantitatively, all these four derivatives are negative, given that the probability of default is increasing in the stock of debt. That is, $\frac{\partial p_U^{ND'}}{\partial B'} \leq 0$; $\frac{\partial q^{ND'}}{\partial B'} \leq 0$; $\frac{\partial p_U^{ND'}}{\partial b'} \leq 0$; and $\frac{\partial q^{ND'}}{\partial b'} \leq 0$.

[52] To avoid using numerical derivatives, instead of using expression (4.4), I use a non-linear solver to find the value of b' for which equation (4.3) holds, for any B' and for the given initial state. It is assumed an initial value of $\bar{y}$ for endowment so that the difference between prices of indexed and non-indexed bonds is not strongly affected by next-period coupons.

Figure 4.8: Rebalancing Portfolios



Notes: Figure shows the change in the total stock of debt required after an issuance of B' units of GDP-linked bonds, so that equation (4.3) holds. It assumes an initial state given by $(y, b, B, \eta) = (\bar{y}, \bar{b}, 0, 0)$, where $\bar{y}$ and $\bar{b}$ represent the unconditional mean of the endowment process and the average stock of non-indexed bonds, respectively. It is assumed that $\bar{c} = \bar{y}$.

this implies that the continuation value is given by $V(y', b(B', X), B', \eta')$. The optimal choice of B' then solves:

$$\begin{gathered} Max_{B'} E_{y'|y} V(y', b(B', X), B', \eta') \\ s.t.\, \eta' = \eta(\eta, B') \\ \tilde{b}(B', X) \geq 0 \end{gathered} \tag{4.5}$$

Totally differentiating equation (4.5) with respect to B' and using the envelope condition on equations (2.6) and (2.7), we get (see Appendix F for the details):

$$\begin{aligned} &\frac{dE_{y'|y} V\left(y', \tilde{b}(B', X), B', \eta'\right)}{dB'} = \\ &E_{y'|y}(1-h')\Bigg\{ -\frac{\partial \tilde{b}(b', X)}{\partial B'} u'(c')\left(\left[z_b + q^{ND''}\right](1-m_b) + m_b\right) + \\ &-u'(c')\left(\left[z_B(y') + p_U^{ND''}\right](1-m_B) + m_B\right) + \frac{\partial V^r\left(y', \tilde{b}(B', X), B', \eta'\right)}{\partial \eta'} \times \frac{\partial \eta'}{\partial B'} \Bigg\} \end{aligned} \tag{4.6}$$

Equation 4.6 represents the expected net marginal benefits of issuing one additional unit of GDP-linked bonds and buying back non-indexed bond with the proceeds of the issuance. In any interior optimum (i.e. with $B' > (1 - m_B) B$ and $B' < \bar{B}$), the previous expression is equal to zero. Hence:

$$\begin{aligned} &E_{y'|y}(1-h')\, u'(c')\left\{ \frac{-\partial \tilde{b}(b', X)}{\partial B'} \left(\left[z_b + q^{ND''}\right](1-m_b) + m_b\right) \right\} = \\ &E_{y'|y}(1-h')\left\{ u'(c')\left(\left[z_B(y') + p_U^{ND''}\right](1-m_B) + m_B\right) - \frac{\partial V^r\left(y', \tilde{b}(B', X), B', \eta'\right)}{\partial \eta'} \times \frac{\partial \eta'}{\partial B'} \right\} \end{aligned} \tag{4.7}$$

The left-hand side of equation (4.7) represents the benefit of issuing one additional unit of GDP-linked bonds. These benefits are given by a decrease in tomorrow's payments of principal and coupons of non-indexed bonds, which allows for an increase in future consumption (conditional on not defaulting). The first term on the right-hand side represents the costs of the swap. They are given by the increase in the future stock of GDP-linked bonds that increases principal and coupon payments (conditional on not defaulting) and therefore depresses future consumption. The last term on the right-hand side captures an additional benefit, given by the fact that an increase in the stock of GDP-linked bonds leads to an increase in next-period measure of constrained investors, decreasing the liquidity premium for future issuances.

From equation (4.7), there are essentially two forces (apart from future prices) behind the choice of the optimal portfolio: (i) the current rate at which non-indexed debt can be replaced with indexed-debt; (ii) the effect of an additional unit of GDP-linked bonds on the liquidity premium.

As shown in Figure 4.8, $-\frac{\partial \tilde{b}(b',X)}{\partial B'} < 1$ when search frictions are considered, which implies that issuing one more unit of GDP-linked bonds is costly as it leads to an increase in the total stock of debt (and hence a decrease in tomorrow's consumption due to the larger coupon and principal payments). As the government is trying to smooth aggregate consumption across time, (i) implies that the government has incentives to spread the issuances of GDP-linked bonds across time in order to spread these costs. However, due to the presence of (ii), the government may still have incentives to issue a large stock of GDP-linked bonds at once in order to decrease the liquidity premium. As shown in Figure (4.1), however, given the calibration of the model, the liquidity premium is almost constant once $B' > 0.4$, implying that the effect of (ii) is negligible for larger values of B'. Overall, these two forces combined help to explain why under search frictions the government introduces GDP-linked bonds at a slower pace, particularly when $\bar{B} = 1.0$.

5 Extension: Income Inequality and Limited Tax Progressivity

In the previous sections, I have focused in only one (second order) benefit of issuing GDP-linked bonds: improving risk sharing between the government and foreign lenders. A key assumption up to this point has been that the sovereign was able to perfectly redistribute the aggregate endowment across the households.[53] Following Ferriere (2015), this section extends the baseline model by considering a set of domestic households that are heterogeneous in their endowments, and by assuming some limitation to the government's capacity to redistribute resources across the domestic households. Under this extension, the benefits of issuing GDP-linked bonds are higher. The intuition behind this result is that GDP-linked bonds decrease the need for procyclical fiscal policies, which are particularly costly in this context given its disproportionate effect on low income households. The goal of this section is to quantify this extra benefits once the liquidity premia is considered.

5.1 The Model

Consider a $[0, 1]$ continuum of household types, indexed by i. Each household i differs in its endowment:

$$y_i = y \times \epsilon_i; \quad \epsilon_i \underset{iid}{\sim} N(1, \sigma_\epsilon) \tag{5.1}$$

Notice that the income distribution $\{y_i\}$ is assumed to be constant across time. Assume that government has a fixed level of public expenditures g, which finances by levying taxes and issuing debt. The main assumption is the following linear tax function:

$$T\left(y_{it}, \tau_1, \tau_{2t}\right) = \tau_1 y_{it} + \tau_{2t} \tag{5.2}$$

where τ_1 measures the degree of progressivity. The parameter τ_1 is assumed to be fixed and smaller than one. In this sense, only τ_{2t} is a choice variable for the government. The linear tax function is an important assumption. With a log-linear tax function and under CRRA preferences, the government chooses identical debt and default policies, independently of the income distribution.

As in the baseline model, it is assumed that households are hand-to-mouth. They value private consumption (c_{it}) and g equally (i.e, they are perfect substitutes). The government is utilitarian and maximizes:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i u(c_{it} + g) d_i \tag{5.3}$$

[53]The assumption is embedded in the use of a representative consumer in the domestic country.

subject to the following budget constraint:

$$
\begin{aligned}
g+b_t\left[(1-m_b)\, z_b+m_b)\right]+B_t\left[(1-m_B)\, z_B\left(y_t\right)+m_B)\right] &= \\
T\left(y_t, \tau_1, \tau_{2,t}\right)+q_t^{ND}\left[b_{t+1}-(1-m_b) b_t\right]+p_{U,t}^{ND}\left[B_{t+1}-(1-m_B) B_t\right] & \qquad (5.4)
\end{aligned}
$$

where:

$$
T\left(y_t, \tau_1, \tau_{2t}\right)=\int T\left(y_{it}, \tau_1, \tau_{2t}\right) di=\tau_1 y_t+\tau_{2t} \tag{5.5}
$$

and subject to each household's budget constraint:

$$
c_{it}=\left(1-\tau_1\right) y_{it}-\tau_{2t} \geq 0 \;\forall i \tag{5.6}
$$

Define aggregate private consumption as $C_t \equiv \int_i c_{it} di$. Integrating over households' budget constraint, we get:

$$
\tau_{2t}=\left(1-\tau_1\right) y_t-C_t \tag{5.7}
$$

Replacing (5.7) in equation (5.5) gives: $T\left(y_t, \tau_1, \tau_{2t}\right)=y_t-C_t$. Finally, replacing (5.7) in the households' budget constraint gives: $c_{it}=\left(1-\tau_1\right)\left(y_{it}-y_t\right)+C_t$. Summing up, as τ_1 is fixed, c_{it} and τ_{2t} are pinned down by C_t. Thus, the government only chooses $\{b', B', d, C\}$ (as in the baseline model). Under CRRA preferences, the government's problem can be summarized as:

$$
Max_{\{C,b,B,d\}_{\forall t}}\; E_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\frac{\left(1-\tau_1\right)\left(y_{it}-y_t\right)+C_t+g}{1-\gamma}\right)^{1-\gamma} di
$$

subject to:

$$
\begin{aligned}
g+C_t=y_t-b_t\left[(1-m_b)\, z_b+m_b)\right]+q_t^{ND}\left[b_{t+1}-(1-m_b) b_t\right]+ \\
-B_t\left[(1-m_B)\, z_B\left(y_t\right)+m_B)\right]+p_{U,t}^{ND}\left[B_{t+1}-(1-m_B) B_t\right] \\
0 \leq \left(1-\tau_1\right)\left(y_{it}-y_t\right) \;\forall i
\end{aligned}
$$

From this last expression, it is easy to see that if taxes were "fully progressive" (that is, $\tau_1=1$) or if there were no income inequality ($y_{it}=y_t$, for all i), then this extension collapses to the baseline model described in Section 2. In other words, as we decrease tax progressivity and increase income inequality, we move further away from the baseline

Table 5.1: Calibration - Extension

Description	Parameter	Value	Targeted Moment
Default Cost	$\{\beta, d_0, d_1\}$	$\{0.978, -1.05, 1.227\}$	2.3% spread and 53% debt/GDP
Income Distribution	$\{\epsilon_i\}_{i=1}^{I=10}$	$\{0.16, 0.22, 0.32, 0.41, 0.52$ $0.83, 0.93, 1.25, 1.89, 3.48\}$	Pre-tax income distribution
Government Spending	$\bar{g}$	0.15	Gov. final consumption expenditure
Tax Progressivity	τ_1	0.4	Post-tax Gini coef.

Notes: This table shows the calibration for the extension of the model.

model.

5.2 Calibration

I calibrate τ_1, g, and $\{\epsilon_i\}_{i=1}^{I}$, in order to match the following moments for Spain (2007-2015): (i) the pre-tax income distribution per decile; (ii) a post-tax Gini index of 0.33; (iii) a general government final consumption expenditure (as a share of GDP) of 15%. The pre-tax and post-tax Gini were retrieved from the OECD. The pre-tax income distribution per decile and the General government final consumption expenditure come from the World Bank. As in Ferriere (2015), I consider an economy populated by 10 different households and choose $\{\epsilon_i\}_{i=1}^{I=10}$ in order to match the pre-tax income distribution.[54] Finally, τ_1 is chosen to match the average after-tax Gini coefficient. I also re-calibrate (β, d_0, d_1) in order to match the targeted moments described in Table 4.2. All other parameters are the same as those described in Table 4.2. The calibration is described in Table 5.1.

5.3 Results: Welfare Gains

Figure 5.1 compares welfare gains after the introduction of GDP-indexed bonds. The top panel shows the results for the case under no-search frictions. Notice that welfare gains increase significantly with respect to the baseline model. For the upper debt limit of $\bar{B} = 1.0$ they are as high as 0.35% in terms of certainty equivalent consumption. Once search frictions are introduced into the model, welfare gains are still reduced in half but they almost double those gains obtained for the baseline model (bottom panel).

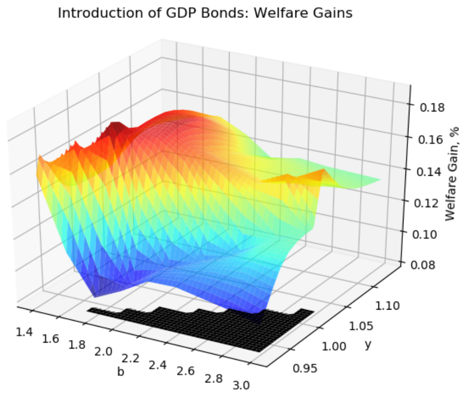
Overall, the results show that after income inequality and limited tax progressivity are incorporated into the model, the government benefits significantly more from introducing

[54] Using the reported $\{\epsilon_i\}_{i=1}^{I=10}$, I obtain a pre-tax Gini Index of 0.48, which is in line with the one reported by the OECD (0.49).

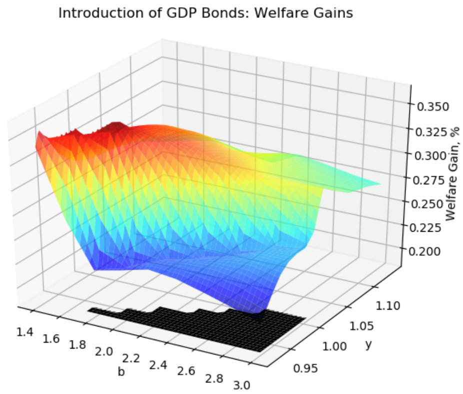
Figure 5.1: Welfare Gains under Heterogeneous Households

No-Search Frictions

$B = 0.4$

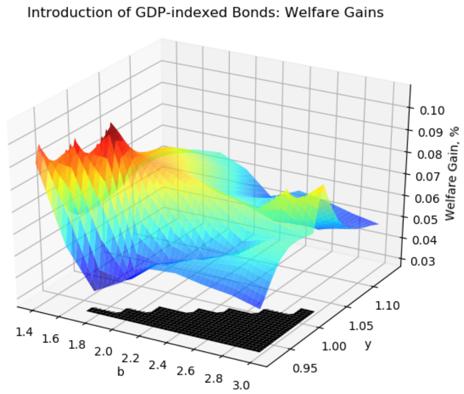


$B = 1.0$

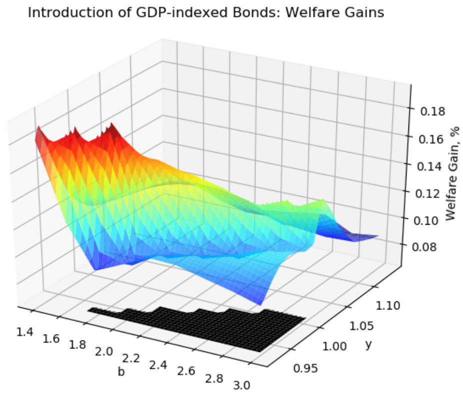


Search Frictions

$B = 0.4$



$B = 1.0$



GDP-linked bonds. In particular, those countries with high income inequality and very regressive tax systems may largely benefit from issuing GDP-indexed bonds. However, as these are features typically observed in less developed countries, it may be the case that these countries also have higher frictions in their secondary markets, which in turn would lead to a larger liquidity premium that prevents them from capturing those higher benefits.

6 Conclusions

Apart from inflation-linked bonds, issuances of state-contingent sovereign debt instruments have been limited both in quantity and frequency. The existing literature has argued that the limited use of these instruments can be explained by the sizable liquidity premium associated with new debt instruments, given the smaller size of their market. In this paper, I quantify how the presence of this liquidity premium erodes the potential benefits associated with the introduction of new types of debt instruments. Although I focus on the case of GDP-linked bonds, the model is flexible enough to accommodate other types of financial instruments. In this regard, this paper can be understood as a general framework to study the limitations of financial innovation in sovereign debt markets.

To account for liquidity risk, I introduce search frictions in the secondary market for GDP-linked bonds. To create a link between the liquidity premium and the size of the secondary market, I allow for free entry of dealers together with an increasing-returns-to-scale matching technology. As the object of interest is the introduction of a new type of instrument, this interaction is key to the study. The model shows that an increase in the stock of GDP-linked bonds increases the number of dealers in the market, leading to a decrease in bid-ask spreads, which in turn leads to a reduction in the liquidity premium demanded at issuance.

In the quantitative analysis, I show that the welfare gains from issuing GDP-linked bonds are significantly affected by the presence of search frictions, particularly when the amount issued of these bonds is small. Overall, they represent only $30-50\%$ of the welfare gains obtained under a no-search frictions scenario. The smaller welfare gains are surprising, since the calibration of the model yields an average liquidity premium of only 35 basis points, which is around the premium observed for the US TIPS, arguably one of the most liquid indexed bonds.

I consider an extension that features (i) income inequality, and (ii) limited tax progressivity. Under this scenario, welfare gains from issuing GDP-linked bonds are significantly higher. The intuition behind this result is that by issuing GDP-indexed bonds, the government reduces the need of implementing procyclical fiscal policies, which are particularly costly in this context given its disproportionate effect on low income households. The key message from this extension is that those countries with higher income inequality and lower tax progressivity are the ones that can benefit the most from issuing these bonds. However, as these two features are typically associated with less developed countries, these larger benefits may be attenuated by a larger liquidity premium.

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A Law of Motion for Measure of Constrained Dealers

This appendix derives the law of motion for the measure of constrained investors η_t; that is, those investors who are currently holding a GDP-linked bond and are liquidity constrained. Let $\tilde{\eta}_t$ denote the measure of unconstrained investors (who are currently holding a GDP-linked bond).

At the beginning of the period, state is $\mathbb{S} = (y, b, B, \eta)$. Government chooses b' and B'. Dealers observe prices and η, and decide to enter the market. The timing assumption is that government's issuances take place at the beginning of period t while trades in the secondary market take place at the end of period t, before the realization of the liquidity shock ζ. Therefore, an investor that holds one unit of a GDP-indexed bond and that suffered from a liquidity shock in period t cannot find a dealer to trade with during period t.

To start, assume that $B_0 = 0$ so that $\eta_0 = 0$, $\tilde{\eta}_0 = 0$, and $\lambda_0 = 0$ as there are no dealers in this market. At the beginning of $t = 0$, government chooses $B_1 = B'(\mathbb{S}_0)$. Hence, our timing assumption implies that: $\eta_1 = \zeta B_1$ and $\tilde{\eta}_1 = (1 - \zeta) B_1$. During $t = 1$, government chooses $B_2 = B'(\mathbb{S}_1)$. The law of motion is:

$$\begin{aligned}
\eta_2 &= \eta_1 (1 - m_B) (1 - \lambda_1) + \underbrace{\tilde{\eta}_1}_{B_1 - \eta_1} (1 - m_B) \zeta + \underbrace{(B_2 - (1 - m_B) B_1)}_{\text{New Issuances}} \zeta \\
&= \eta_1 (1 - m_B) (1 - \lambda_1 - \zeta) + B_2 \zeta
\end{aligned}$$

Notice that:

$$\begin{aligned}
\tilde{\eta}_2 &= \eta_1 (1 - m_B) \lambda_1 + \tilde{\eta}_1 (1 - m_B) (1 - \zeta) + (B_2 - (1 - m_B) B_1) (1 - \zeta) \\
&= \tilde{\eta}_1 (1 - m_B) [1 - \lambda_1 - \zeta] - B_1 (1 - m_B) [1 - \lambda_1 - \zeta] + B_2 (1 - \zeta)
\end{aligned}$$

Therefore, combining the previous two expressions, notice that: $\eta_2 + \tilde{\eta}_2 = B_2$. In general, we have that:

$$\begin{aligned}
\eta_{t+1} &= \eta_t (1 - m_B) (1 - \lambda_t) + \tilde{\eta}_t (1 - m_B) \zeta + (B_{t+1} - (1 - m_B) B_t) \zeta \\
&= \eta_t (1 - m_B) (1 - \lambda_t - \zeta) + B_{t+1} \zeta \qquad \text{(A.1)}
\end{aligned}$$

Notice that while in default, $B_{t+1} = B_t$ because the government is not issuing new debt and the maturity of bonds remains constant (i.e. $m_B = 0$ while in default). Thus, the law of motion is:

$$\eta_{t+1} = \eta_t \left(1 - \lambda_t - \zeta\right) + B_t \zeta \tag{A.2}$$

B Random Search and IRS Matching Technology

Based on DGP (2005), this appendix provides micro-foundations for the use of an increasing-returns-to-scale (IRS) matching technology. This assumption seems realistic for financial markets because it implies that investors can find a trading partner more easily in larger markets (see for instance, Vayanos and Wang, 2007 or Geromichalos and Herrenbrueck, 2020).

Consider the following general scenario. Let Δ denote the period length. There are two types of dealers: active and non-active. Let A denote the subset of active dealers and $\tilde{A}$ denote the subset of those that are not active. In this context, an active dealer should be interpreted merely as an investor who is currently not holding an state-contingent bonds and would like to buy one unit.[55] On the other hand, as described in Section 2, there are two types of investors that are currently holding a state-contingent bond: constrained investors (i.e., sellers) and unconstrained investors. Let C denote the subset of constrained investors; and $\tilde{C}$, denote the subset of unconstrained investors.[56] Finally, let $N =\mid A \mid + \mid \tilde{A} \mid + \mid C \mid + \mid \tilde{C} \mid$ be the total population.

Search frictions are only considered on the sell-side of the secondary market. To keep things simple, assume that with probability $p\left(N, \Delta\right) = 1 - e^{-\frac{\Delta}{N}\lambda}$ (that is, the probability of a Poisson process with intensity λ/N), agent i contacts agent j, chosen from the entire population *at random*. The rationale behind this random search procedure is that agents cannot tell before contacting a counterparty whether she is a dealer (active or not active) or an investor. Only when a constrained investor meets an active dealer a trade will occur. Figure B.1 summarizes the different meetings that may emerge under random matching. Solid lines represent meetings that will lead to trade. Dashed lines represent meetings that will lead to no trade.

[55] In the terms of DGP (2005), these would be high valuation investors who are currently not holding the asset. On the other hand, non-active dealers are low valuation investors who are not holding the asset.

[56] To be clear, in Section 2, η refers to the *measure* of constrained investors, as a function of B, which in turns is expressed as a share of GDP. In this appendix, C is simply capturing the *number* of constrained investors.

Let $\mathbb{I}_{i,j}$ denote a contact of agent j by agent i. Assume $\mathbb{I}_{i,j}$ is independent across all distinct pairs (i,j) of distinct agents. Under this assumption, the mean rate of contact per unit of time is:

$$E\left[\frac{1}{\Delta}\sum_{i\neq j}\mathbb{I}_{i,j}\right]=\frac{1}{\Delta}(N-1)\times p\left(N,\Delta\right)$$

Taking limits, by L'Hopital rule:

$$\begin{aligned}lim_{(N,\Delta)\to(\infty,0)}E\left[\frac{1}{\Delta}\sum_{i\neq j}\mathbb{I}_{i,j}\right]&=lim_{(N,\Delta)\to(\infty,0)}\left[\frac{1}{\Delta}(N-1)\times p\left(N,\Delta\right)\right]\\&=lim_{(N,\Delta)\to(\infty,0)}\frac{N-1}{\Delta}\left[1-e^{-\Delta\lambda\frac{1}{N}}\right]\\&=\lambda\end{aligned}$$

We are interested in the meetings between the subset of constrained investors $C(N)$ and the (disjoint) subset of active dealers $A(N)$, as these meetings will lead to trade. The per-capita total rate of contact (per unit of time) by the subset $C\left(N\right)$ with subset $A\left(N\right)$ is:

$$S^{C,A}\left(N,\Delta\right)=\frac{1}{N\Delta}\left[\sum_{i\in C(N),j\in A(N)}\mathbb{I}_{i,j}+\sum_{i\in A(N),j\in C(N)}\mathbb{I}_{i,j}\right]$$

Taking expectations:

$$\begin{aligned}E\left[S^{C,A}\left(N,\Delta\right)\right]&=\frac{1}{N\Delta}\left\{p\left(N,\Delta\right)\mid C\mid\mid A\mid+p\left(N,\Delta\right)\mid A\mid\mid C\mid\right\}\\&=\frac{2}{N\Delta}p\left(N,\Delta\right)\mid C\mid\mid A\mid\end{aligned}$$

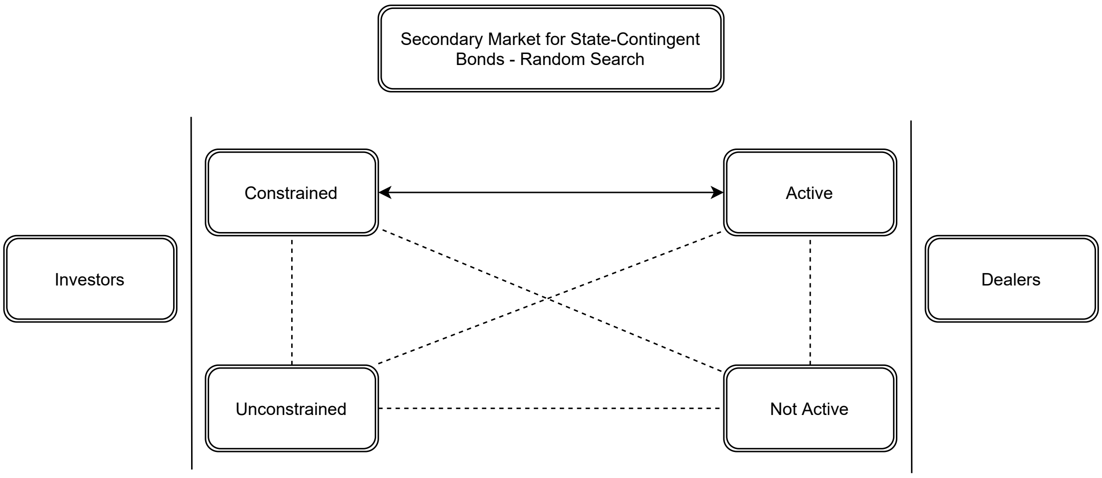
Define $\mu_C=\frac{|C|}{N}$ and $\mu_A=\frac{|A|}{N}$, as the share of constrained investors and active dealers as a fraction of the total population N. Taking limits, we obtain that the (expected) number of matches between constrained sellers and active dealers is:

$$\begin{aligned}lim_{(N,\Delta)\to(\infty,0)}E\left[S^{C,A}\left(N,\Delta\right)\right]&=2\lambda\mu_C\mu_A\\&\equiv M\left(\mu_C,\mu_A\right)\end{aligned}\tag{B.1}$$

As discussed by DGP (2005), one caveat that arises in discrete time, is that an agent can contact more than one other agent at the same time. In that case, it may be possible to set an elimination rule in order to keep only one-to-one matches. However, since the probability of contacting more than one agent during a period of length Δ is of the order Δ^2, the meeting rate is as derived above.

Equation (B.1) shows that under random matching, the function $M\left(\mu_C,\mu_A\right)$ presents increasing returns to scale. Based on this, the baseline model presented in Section 2

Figure B.1: Secondary Market for State-Contingent Bonds - Matching Technology



Notes: Figure describes the matching technology for the secondary market of state-contingent bonds. Black solid lines represent a meeting between a constrained investor that is willing to sell and an active dealer that is willing to buy. Thus, solid lines represent a meeting that leads to trade. Dashed lines represent those cases in which the meeting does not lead to trade.

assumes a (more general) IRS matching technology given by: $m\left(\mu_C, \mu_A\right) = \left[\mu_A^{\chi_1} \mu_C^{(1-\chi_1)}\right]^{\chi_2}$ where $\chi_2 > 1$, to better capture the targeted moments described in the calibration section.

We can generalize the previous result for the case in which investors do not meet between each other and neither do dealers. With probability p_{ij} an investor i contacts dealer j, chosen from the population of dealers (not entire population) *at random.* On the other hand, with probability p_{ji} a dealer j contacts investor i, chosen from the population of investors *at random.* Let $D =\mid A \mid + \mid \tilde{A} \mid$ denote the number of dealers and $I =\mid C \mid + \mid \tilde{C} \mid$ denote the number of investors. Under this assumption, we can write:

$$p_{ij}\left(I, D, \Delta\right) = \begin{cases} 1 - e^{-\frac{\Delta}{D}\lambda} & \text{if } i \in \left(C \cup \tilde{C}\right), j \in \left(A \cup \tilde{A}\right) \\ 1 - e^{-\frac{\Delta}{I}\lambda} & \text{if } i \in \left(A \cup \tilde{A}\right), j \in \left(C \cup \tilde{C}\right) \\ 0 & \text{otherwhise} \end{cases}$$

Let $D = f\left(I\right)$ denote the relation between the sizes of the two populations. In the model presented in Section 2, this relation is endogenous and it's pin-down through the dealers' free-entry condition. Take this relation as given here and assume that $lim_{I\to\infty} f(I) = \infty$. The per-investor total rate of contact (per unit of time) by the subset $C\left(I\right)$ with subset $A\left(I\right)$is:

$$S^{C,A}\left(I, \Delta\right) = \frac{1}{I\Delta}\left[\sum_{i\in C(I), j\in A(I)} \mathbb{I}_{i,j} + \sum_{i\in A(I), j\in C(I)} \mathbb{I}_{i,j}\right]$$

Let $\mu_C = \frac{|C|}{I}$ and $\mu_A = \frac{|A|}{D}$. It is easy to show that:

$$\begin{aligned} lim_{(I,\Delta)\to(\infty,0)} E\left[S^{C,A}(I,\Delta)\right] &= \lambda\mu_C\mu_A + \mu_C\mu_A lim_{(I,\Delta)\to(\infty,0)} \frac{f(I)}{\Delta}\left(1 - e^{-\frac{\Delta}{I}\lambda}\right) \\ &= \lambda\mu_C\mu_A + \lambda\mu_C\mu_A lim_{(I,\Delta)\to(\infty,0)} \left[\frac{f(I)}{I}\right]^2 \frac{1}{f'(I)} \end{aligned}$$

A sufficient condition for the limit on the right-hand side to exist is $f(I)$ to be linear, implying that dealers' population increases at the same rate as investors' population. Assuming $f(I) = \phi I$, we get:

$$\begin{aligned} lim_{(I,\Delta)\to(\infty,0)} E\left[S^{C,A}(I,\Delta)\right] &= \lambda(1+\phi)\mu_C\mu_A \\ &\equiv M(\mu_C,\mu_A) \end{aligned}$$

C Computational Algorithm

The algorithm iterates on three value functions, $V(.), V^r(.), V^d(.)$, and on six prices $q^{ND}(.), q^D(.)$, and $p_U^{ND}(.), p_C^{ND}(.), p_U^D(.), p_C^D(.)$ until convergence is attained. I approximate the value functions and the price schedules using linear interpolation, using the "Interpolations" package in Julia. The endowment process is discretized using Tauchen's (1986) method. Grids of evenly distributed points are constructed for the four state variables (y, b, B, η). I use 19 points for y, 21 points for b and 11 points for B and η.

The following algorithm was used to solve the model:

1. Start with some guess for the value functions V, V^r, V^d and for the prices q^{ND}, q^D, p_U^{ND}, p_C^{ND}, p_U^D, p_C^D.
2. Using the prices of step (1), for each point of the state-space, solve for the number of dealers $v^\star = v(y, b, B, \eta)$ that satisfy the free-entry condition defined on equation (2.4). Compute the probability of finding a dealer $\lambda(.)$.
3. Solve the optimization problem defined in equation (2.6). In order to solve the portfolio problem related to the choice of (b', B'), I employ the following routine:
    - (i) For each point of the state-space (y, b, B, η), search over $\bar{X}$ points for b' and over $\bar{Y}$ points for B', where $\bar{X} = 21 + 40$ and $\bar{Y} = 11 + \lfloor 40(\bar{B} - B) \rfloor$. The number of $\bar{Y}$ points is a consequence of the no-buy-back assumption regarding state-contingent bonds. Under this assumption, it is possible to decrease the searching grid size as B increases, obtaining significant speed gains.[57]

[57]For the model with heterogeneous agents and limited tax progressivity, I employ more points because

- (ii) Second, using the solution of step (i), keep fix $(b' + B')$, and search over 150 points for B'.
- (iii) Third, using the solution of step (ii) as a guess, implement a two-dimensional local maximum algorithm. In particular, I use the "SBPLX" algorithm of the "NLOpt" Julia's package.[58]

4. Using the optimal solution for (b', B') computed in Step 3, update $V^r(.)$ as defined in equation (2.6). Update $V^d(.)$ as defined in equation (2.7) and compute $V(.) = max\left\{V^r(.), V^d(.)\right\}$, together with the optimal default decisions $h\left(.\right)$.
5. Using the optimal solution for (b', B') and the optimal default decisions $h\left(\mathbb{S}\right)$, update prices using equations (2.8)-(2.13).
6. If the maximum distance between the updated values for V, V^r, V^d and for the six price schedules q^{ND}, q^{D}, p_U^{ND},$p_C^{ND}, p_U^{D}, p_C^{D}$ and their previous ones is below 10^{-5}, stop the algorithm. Otherwise, update value functions and prices using a dampening coefficient of 0.95 and go back to step (2).

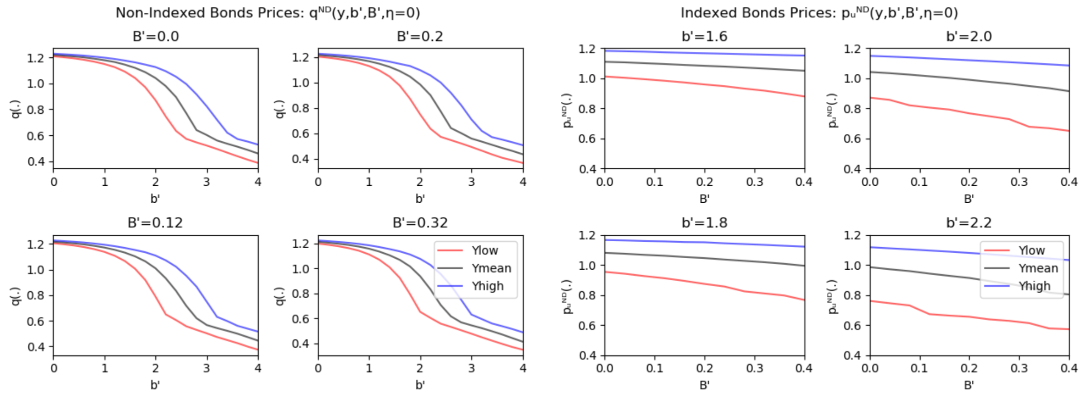
D Additional Figures - Baseline Model

This appendix presents additional figures of the main quantitative analysis. Figure D.1 presents the pricing kernels for different combinations of (b', B'). Figure D.2 depicts the probability of finding a dealer for different sizes of the secondary market, η. Lastly, Figure D.3 shows the bid-ask spreads and liquidity premium for the case in which $\bar{B} = 1.0$.

the value function presents more kinks.

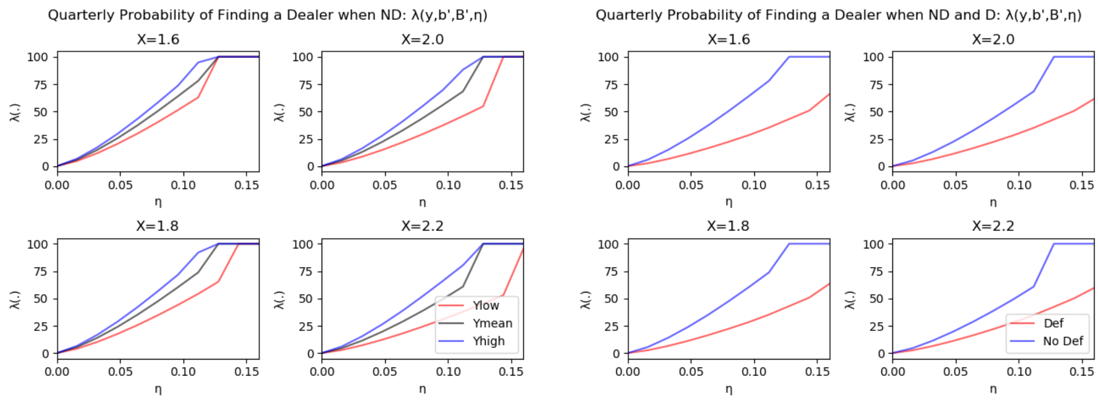
[58]Subplex is a variant of the popular Nelder-Mead algorithm that uses Nelder-Mead on a sequence of subspaces. It is claimed to be more efficient and robust than the original Nelder-Mead, while retaining the latter's facility with discontinuous objectives. See https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#sbplx-based-on-subplex for further reference.

Figure D.1: Pricing Kernels



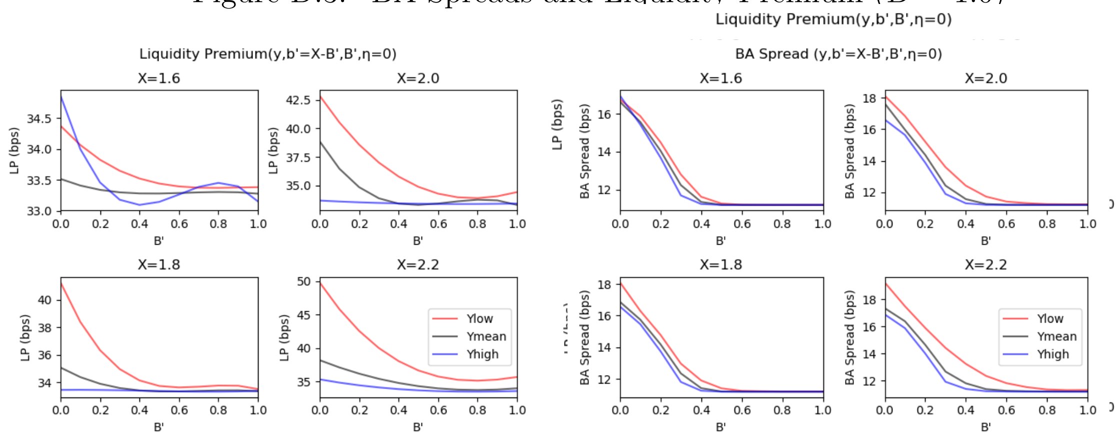
Notes: Prices at issuance of non-indexed bonds -$q^{ND}(.)$-and GDP-linked bonds -$p_U^{ND}(.)$- for different issuances of b' and B'. In all cases, it is assumed that $(B, \eta) = (0, 0)$. $y_{Low} = \bar{y} - \sigma_y$ and $y_{High} = \bar{y} + \sigma^y$, where σ^y is computed after simulating the economy 10,000 times and dropping those paths were the sovereign government defaulted (see section 4.2 for additional details). Figures show the case for the debt limit of $\bar{B} = 0.4$ (10% of annual GDP).

Figure D.2: Probability of Finding a Dealer



Notes: Prices at issuance of non-indexed bonds -$q^{ND}(.)$-and GDP-linked bonds -$p_U^{ND}(.)$- for different issuances of b' and B'. In all cases, it is assumed that $(B, \eta) = (0, 0)$. $y_{Low} = \bar{y} - \sigma_y$ and $y_{High} = \bar{y} + \sigma^y$, where σ^y is computed after simulating the economy 10,000 times and dropping those paths were the sovereign government defaulted (see section 4.2 for additional details). Results for the debt limit of $\bar{B} = 0.4$ (10% of annual GDP).

Figure D.3: BA Spreads and Liquidity Premium ($\bar{B} = 1.0$)



Notes: Liquidity premium (left panel) and bid-ask spreads (right panel) for GDP-linked bonds for different combinations of the state space. The initial portfolio is given by $(b, B) = (X, 0)$ and, in all cases, the next-period total stock of debt (i.e., $b' + B'$) is held constant at X. $y_{Low} = \bar{y} - \sigma_y$ and $y_{High} = \bar{y} + \sigma^y$, where σ^y is computed after simulating the economy 10,000 times and dropping those paths were the sovereign government defaulted (see section 4.2 for additional details). The liquidity premium was smoothed using a third order degree polynomial to avoid some small kinks. Figures correspond to the debt limit of $\bar{B} = 1.0$ (25% of annual GDP).

E Results under a Constant-Returns-to-Scale Matching Function

The following appendix analyzes the baseline model presented in Section 2, under the assumption of a constant-returns-to-scale (CRS) matching technology.

E.1 Overview

Constant-Returns-to-Scale Matching Technology

Let $\tilde{m}(v,\eta) = v^{\chi_1} \times \eta^{(1-\chi_1)}$ denote a CRS matching technology. Assume that $\chi_1 < 1$ and, for simplicity, that $m(v,\eta) < min(v,\eta)$. Let $\mathbb{S} = (y, b', B', \eta)$ be the state after the government chooses its new bond policies. From the dealers' free-entry condition we have that:

$$v(\mathbb{S}) = \eta \left[\frac{(1-\alpha)\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]}{k} \right]^{\frac{1}{1-\chi_1}} \tag{E.1}$$

By substituting equation (E.1) into the matching function $\tilde{m}(v,\eta)$, we can compute the probability for a constrained investor to find a dealer as:

$$\begin{aligned} \lambda(\mathbb{S}) &= \frac{v^{\chi_1}\eta^{(1-\chi_1)}}{\eta} \\ &= \left\{ \eta \left[\frac{(1-\alpha)\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]}{k} \right]^{\frac{1}{1-\chi_1}} \right\}^{\chi_1} \eta^{-\chi_1} \\ &= \left[\frac{(1-\alpha)\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]}{k} \right]^{\frac{\chi_1}{1-\chi_1}} \end{aligned} \tag{E.2}$$

These two expressions highlight the fact that under CRS the only thing that matters is the tightness ratio $\vartheta \equiv \frac{v}{\eta}$. Assume that there is an increase in η, while all the other states remain the same. According to equation (E.1), for a given price difference $\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right] > 0$, new dealers will enter the market. In particular, dealers will enter in a way such that the price difference $\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]$ remains constant, so that ϑ and $\lambda(\mathbb{S})$ also remain constant. This is depicted in Figure E.1. The left-hand side panel plots $\lambda(\mathbb{S})$ for different values of η, keeping the other three state variables constant. The right-hand side panel plots the price difference $\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]$. Notice that both $\lambda(\mathbb{S})$ and the price difference are constant and they do not depend on the value of η.

To conclude, even when $\lambda(\mathbb{S})$ does not depend directly on η, it does depend on the stock of state-contingent bonds. In other words, a change in the government's portfolio

(b, B) affects $\lambda(\mathbb{S})$ because it affects the sovereign's default incentives and therefore its prices. These changes, however, are significantly lower than those observed under IRS.

Increasing-Returns-to-Scale Matching Technology

Let $m(v, \eta) = \left[v^{\chi_1} \times \eta^{(1-\chi_1)}\right]^{\chi_2}$ be the matching technology. Assume that $\chi_1 < 1$, $\chi_2 > 1$ and define $\chi = \chi_1 \chi_2$. For simplicity, assume that $m(v, \eta) < min(v, \eta)$. From the dealer's free-entry condition we have that:

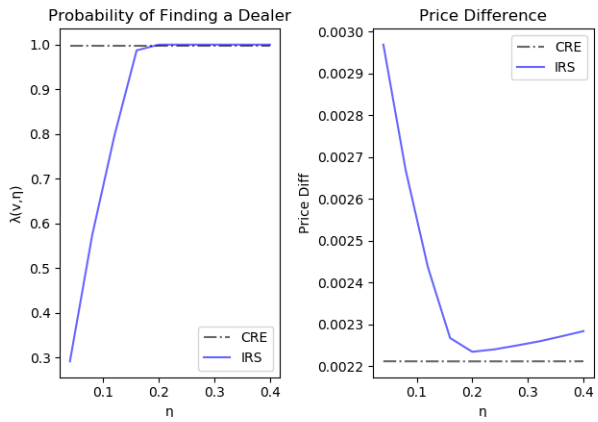
$$v = \eta^{\left(\frac{\chi_2 - \chi}{1-\chi}\right)} \left[\frac{(1-\alpha)\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]}{k}\right]^{\frac{1}{1-\chi}} \tag{E.3}$$

After substituting this expression in the matching function, we get that the probability of finding a dealer is given by:[59]

$$\lambda(\mathbb{S}) = \eta^{\frac{\chi_2 - 1}{1-\chi}} \left[\frac{(1-\alpha)\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]}{k}\right]^{\frac{\chi}{1-\chi}} \tag{E.4}$$

Figure E.1 (blue-solid line) plots this probability under the IRS matching technology, for different values of η, keeping constant the other three state variables. Unlike the CRS case, notice that $\lambda(\mathbb{S})$ is increasing in η, reflecting the fact that as the number of sellers in this market increases, it is easier for them to find a dealer to trade with.

Figure E.1: Probability of Finding a Dealer as function of η



Notes: Left panel shows the probability of finding a dealer as a function of the measure of constrained investors η. Right panel shows the price difference $p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})$. The other three state variables are fixed constant at: $(y, b', B') = (1., 1.8, 0.4)$. Only the upper debt limit case $\bar{B} = 1.0$ is considered.

E.2. Analysis of $\lambda(\mathbb{S})$ under a CRS Matching Technology

This subsection informally shows that, under a CRS matching technology, $\lambda(\mathbb{S})$ is constant in η. First, I start by showing that there exists a function $\lambda(\mathbb{S})$ that is constant

[59] Of course, if $\chi_2 = 1$, then we get the same expression as with CRS.

in η that satisfies equation (E.2), given the equilibrium pricing kernels. Then, I discuss that any other function $\lambda(\mathbb{S})$ that is not constant in η, cannot be an equilibrium outcome.

Consider, for simplicity, the case with $f = 0$ (so there is no debt recovery). With a slight change of notation from the pricing kernels of Section 2, we have that:

$$p_U^{ND}(\mathbb{S}) \times (1 + r_U) = \int \{[1 - h(\mathbb{S}')] \left[m_B + (1 - m_B)\left[z_B(y') + \zeta p_C^{ND}(\mathbb{S}') + (1 - \zeta) p_U^{ND}(\mathbb{S}')\right]\right]\} dF(y' \mid y) \tag{E.5}$$

$$p_C^{ND}(\mathbb{S}) \times (1 + r_C) = \int \{[1 - h(\mathbb{S}')] \left[m_B + (1 - m_B)\left[z_B(y') + [1 - \lambda(\mathbb{S}')] p_C^{ND}(\mathbb{S}') + \lambda(\mathbb{S}') p_B^{ND}(\mathbb{S}')\right]\right]\} dF(y' \mid y) \tag{E.6}$$

where $\mathbb{S} = (y, b', B', \eta)$, $\mathbb{S}' = (y', b'', B'', \eta')$, $b'' = b(\mathbb{S})$, $B'' = B(\mathbb{S})$, and $\eta' = [\eta(1 - m_B)(1 - \lambda(\mathbb{S}) - \zeta) + B'\zeta]$.[60]

Combining these two pricing kernels together with equation (E.2) we have:

$$\begin{aligned}(1 + r_U) p_U^{ND}(\mathbb{S}) - (1 + r_C) p_C^{ND}(\mathbb{S}) &= \\ (1 - m_B)\frac{\kappa}{1 - \alpha} \int &\left\{ [1 - h(\mathbb{S}')] \{1 - \zeta - \alpha\lambda(\mathbb{S}')\} \lambda(\mathbb{S}')^{\frac{1-\chi_1}{\chi_1}} \right\} dF(y' \mid y)\end{aligned}$$

Notice that the left-hand side of the previous equation can be written as:

$$(1 + r_U)\left[\lambda(\mathbb{S})^{\frac{1-\chi_1}{\chi_1}} \frac{\kappa}{1 - \alpha} + \underbrace{\left[\frac{(1 + r_C)}{1 + r_U} - 1\right]}_{\approx 0} p_C^{ND}(\mathbb{S})\right]$$

where the last term will be omitted from here on as it is almost zero. Summing up, to be a solution, the functional form $\lambda(.)$ must satisfy that:

$$(1 + r_U)\lambda(\mathbb{S})^{\frac{1-\chi_1}{\chi_1}} = (1 - m_B)\int [1 - h(\mathbb{S}')] \{1 - \zeta - \alpha\lambda(\mathbb{S}')\} \lambda(\mathbb{S}')^{\frac{1-\chi_1}{\chi_1}} dF(y' \mid y) \tag{E.7}$$

Next, I show that any function $\lambda(.)$ that is constant in η is a solution to equation (E.7).

(1) Remind that prices are a function of: (i) the probability of default; (ii) the probability of finding a dealer.[61] If both are kept constant, prices do not change (assuming that the initial state does not change). With this in mind, conjecture that the probability of finding a dealer is constant in η. Assume an increase in η (i.e, $\tilde{\eta} > \eta$). Let $\mathbb{S} = (y, b', B', \eta)$ and $\tilde{\mathbb{S}} = \left(y, \tilde{b}', \tilde{B}', \tilde{\eta}\right)$, where bond policies are a function of the initial state (y, b, B, η)

[60] Remember that the bid price is: $p_B^{ND}(\mathbb{S}) = (1 - \alpha) p_C^{ND}(\mathbb{S}) + \alpha p_U^{ND}(\mathbb{S})$

[61] Indirectly, prices are also affected by the bond policies through their effect on the default probability and on the probability of finding a dealer.

and $(y, b, B, \tilde{\eta})$, respectively.

(2) Under our conjecture, bond and default policies have to be unaffected by changes in η . To see why, assume that bond policies depend on η. In this case, it is then possible to choose a pair $(\eta, \tilde{\eta})$ such that: $(b', B') \neq \left(\tilde{b}', \tilde{B}'\right)$. The different next-period initial state implies a change in tomorrow's default probability, which in turn leads to a change in current prices. In other words, prices depend on η. However, equation (E.2) shows that different prices lead to different $\lambda(.)$, violating our conjecture. Thus, under our conjecture, $(b', B') = \left(\tilde{b}', \tilde{B}'\right)$, implying that $\lambda(\tilde{\mathbb{S}}) = \lambda(\mathbb{S})$. Same analysis can be done for the next-period default policies and we get that: $h' = \tilde{h}$'.

(3) Given that bond policies are not affected by η, then, following the same analysis as in (2): $(b'', B'') = \left(\tilde{b}'', \tilde{B}''\right)$. Thus, under our conjecture that $\lambda(.)$ does not depend on η: $\lambda(\tilde{\mathbb{S}}') = \lambda(\mathbb{S}')$ and equation (E.7) holds (given that $h' = \tilde{h}'$).

In what follows, I informally discuss that any function $\lambda(\mathbb{S})$ that is not constant in η, cannot be an equilibrium outcome. Consider any other functional form for $\lambda(\mathbb{S})$ that depends on the state η. To start, conjecture that $\lambda(\mathbb{S})$ is increasing in η. Based on equation (E.2), it must then be the case that the difference $\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]$ is increasing in η. However, notice from equations (E.5) and (E.6) that $p_C^{ND}(\mathbb{S})$ depends positively on $\lambda(\mathbb{S})$, while $p_U^{ND}(\mathbb{S})$ only depends indirectly (positively) on $\lambda(\mathbb{S})$ through its effects on $p_C^{ND}(\mathbb{S}')$. Therefore, $\frac{\partial p_C^{ND}(\mathbb{S})}{\partial \eta} > \frac{\partial p_U^{ND}(\mathbb{S})}{\partial \eta} > 0$, which implies that the difference $\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]$ is decreasing in η , contradicting our initial conjecture.[62]

On the other hand, if $\lambda(\mathbb{S})$ were decreasing in η, from equation (E.2) it must then be the case that $\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]$ is decreasing in η. However, from equations (E.5) and (E.6) now we have that: $\frac{\partial p_C^{ND}}{\partial \eta} < \frac{\partial p_U^{ND}}{\partial \eta} < 0$, which implies that the difference $\left[p_U^{ND}(\mathbb{S}) - p_C^{ND}(\mathbb{S})\right]$ is increasing in η, contradicting our initial conjecture.

E.3. Comparison of Welfare

For this part of the appendix, I solve the baseline model described in Section 2, under the assumption of CRS. Importantly, even when $\lambda(\mathbb{S})$ is not a function of η, it is still a function of the stock of state-contingent bonds and on the endowment y. This is because a change in the composition of the portfolio (b, B) or a change in current endowment

[62]This informal analysis assumes that a change in η does not generate a significant change (if any) in the default probability.

Table E.2: Welfare Gains - IRS vs CRS Comparison

	$\bar{B} = 0.4$			$\bar{B} = 1.0$		
	IRS	CRS	Ratio	IRS	CRS	Ratio
$\bar{y} - 2\sigma_y$	0.027	0.031	87%	0.090	0.090	100%
$\bar{y}$	0.028	0.032	87%	0.100	0.102	98%
$\bar{y} + 2\sigma_y$	0.021	0.022	95%	0.075	0.077	97%

Notes: Comparison of welfare gains under increasing returns to scale (IRS) and under constant returns to scale (CRS). The Ratio columns display the IRS welfare gains divided by the CRS welfare gain. $\bar{y}$ represents the unconditional mean of the income porcess. The table assumes a level of non-indexed bonds equal to $\bar{b}$ (i.e., the average amount of non-indexed bonds before indexed debt is introduced).

does have an impact on default policies and therefore on prices, affecting $\lambda(\mathbb{S})$, according to expression (E.2). Therefore, the results under a CRS matching technology are not the same as those obtained in a model in which λ is exogenous and constant, as in Passadore and Xu (2020).

Table E.1 describes the calibration. All other parameters are left the same. Under this calibration, the CRS model is able to replicate the 10 bps BA spread targeted in the baseline model. However, as the BA spread is invariant to changes in η, it cannot replicate the larger BA spread targeted when the size of the state-contingent bonds' secondary market is small.[63]

Table E.1: Calibration - CRS Matching Function

Description	Parameter	Value
Matching Parameter	$\{\chi_1, \chi_2\}$	$\{0.5, 1.0\}$

Table E.2 compares the welfare gains under an IRS matching function (same as those in Section 2) and under CRS. It assumes a level of non-indexed bonds given by $\bar{b}$ (i.e., the average amount of non-indexed bonds before state-contingent debt is introduced) and three different levels of income ($\bar{y} \pm 2\sigma_y$). Notice that welfare gains are almost identical in the two cases. Overall, gains are slightly larger (around 10%) for the case in which $\bar{B} = 0.4$ under CRS due to a lower liquidity premia for low levels of η. In other words, under CRS, the government does not have to pay an extra premium for introducing this type of bonds. For the larger debt limit, welfare gains are almost identical under the two different matching technologies.

[63]In fact, the BA spreads are flat at 10 bps for any point of the state space (as long as $\eta > 0$).

F Further Details on Section

To derive equation (4.7) in section 4.4, we use the envelope theorem on equations (2.6) and (2.7), to get:[64]

$$
\begin{aligned}
\frac{\partial V^r}{\partial b} &= -u'(c) \times \left\{ \left[\left(1-m_b\right) z_b + m_b \right) \right] + q^{ND'} \left(1-m_b\right) \right\} \\
&= -u'(c) \times \left\{ \left[z_b + q^{ND'} \right] \left(1-m_b\right) + m_b \right) \right\}
\end{aligned} \tag{G.1}
$$

$$
\begin{aligned}
\frac{\partial V^r}{\partial B} &= -u'(c) \times \left\{ \left[\left(1-m_B\right) z_B\left(y\right) + m_B \right) \right] + q^{ND'} \left(1-m_B\right) \right\} \\
&= -u'(c) \times \left\{ \left[z_B\left(y\right) + p_U^{ND} \right] \left(1-m_B\right) + m_B \right) \right\}
\end{aligned} \tag{G.2}
$$

Finally, given the assumption that $f = 0$:

$$
\frac{\partial V^d}{\partial b} = 0 \tag{G.3}
$$

$$
\frac{\partial V^d}{\partial B} = 0 \tag{G.4}
$$

Differentiating 4.5, we get:

$$
\begin{aligned}
\frac{dE_{y'|y}V\left(y', \tilde{b}\left(B', X\right), B', \eta'\right)}{dB'} &= \frac{\partial \tilde{b}\left(B', X\right)}{\partial B'} \frac{\partial E_{y'|y}V\left(y', \tilde{b}\left(B', X\right), B', \eta'\right)}{\partial b'} + \\
&+ \frac{\partial E_{y'|y}V\left(y', \tilde{b}\left(B', X\right), B', \eta'\right)}{\partial B'} + \frac{\partial E_{y'|y}V\left(y', \tilde{b}\left(B', X\right), B', \eta'\right)}{\partial \eta'} \times \frac{\partial \eta'}{\partial B'}
\end{aligned} \tag{G.5}
$$

Using (G.1)-(G.4) and the fact that:

$$
V(y, b, B, \eta) = Max_{d\{0,1\}} \left\{ V^d\left(y, b, B, \eta\right), \, V^r\left(y, b, B, \eta\right) \right\}
$$

we obtain:

$$
\frac{\partial E_{y'|y}V\left(y', \tilde{b}\left(B', X\right), B', \eta'\right)}{\partial b'} = -E_{y'|y}\left(1-h'\right) \left\{ u'(c') \times \left\{ \left[z_b + q^{ND''} \right] \left(1-m_b\right) + m_b \right\} \right\}
$$

$$
\frac{\partial E_{y'|y}V\left(y', \tilde{b}\left(B', X\right), B', \eta'\right)}{\partial B'} = -E_{y'|y}\left(1-h'\right) \left\{ u'(c') \times \left\{ \left[z_B\left(y'\right) + p_U^{ND''} \right] \left(1-m_B\right) + m_B \right\} \right\}
$$

$$
\frac{\partial E_{y'|y}V\left(y', \tilde{b}\left(B', X\right), B', \eta'\right)}{\partial \eta'} = E_{y'|y}\left(1-h'\right) \left\{ \frac{\partial V^r\left(y', \tilde{b}\left(B', X\right), B', \eta'\right)}{\partial \eta'} \right\}
$$

Replacing these three expressions on equation (G.5), we obtain equation (4.6) of the

[64]For simplicity, I assume that none of the inequality constraints are binding.

main text:

$$\frac{dE_{y'|y}V\left(y',\tilde{b}\left(B',X\right),B',\eta'\right)}{dB'} = E_{y'|y}\left(1-h'\right)\Bigg\{-\frac{\partial\tilde{b}\left(b',X\right)}{\partial B'}u'(c')\left(\left[z_b+q^{ND''}\right](1-m_b)+m_b\right)+ \tag{G.6}$$

$$-u'(c')\left(\left[z_B\left(y'\right)+p_U^{ND''}\right](1-m_B)+m_B\right)+\frac{\partial V^r\left(y',\tilde{b}\left(B',X\right),B',\eta'\right)}{\partial\eta'}\times\frac{\partial\eta'}{\partial B'}\Bigg\}$$

Euler Equations Analysis

An alternative way to derive equation (4.7) uses the Euler equations for both b' and B'. For simplicity, assume that $\frac{\partial\eta'}{\partial b'}=0$.[65] The Lagrangian of the problem is given by:[66]

$$L = u(c)+\beta\int_{y'}V\left(y',b',B',\eta'\right)dF\left(y'\mid y\right)+\lambda_1\left[\bar{B}-B'\right]+\lambda_2\left[B'-\left(1-m_B\right)B\right]$$

Taking the first order condition with respect to b':

$$u'(c)\times\left[q^{ND}+\frac{\partial q^{ND'}}{\partial b'}\left[b'-(1-m_b)b\right]+\frac{\partial p_U^{ND'}}{\partial b'}\left[B'-(1-m_B)B\right]\right]+\beta\frac{\partial E_{y'|y}V\left(y',b',B',\eta'\right)}{\partial b'}=0$$

Substituting (G.1)-(G.4) the previous expression, we get:

$$u'(c)\times\left[q^{ND'}+\frac{\partial q^{ND'}}{\partial b'}\left[b'-(1-m_b)b\right]+\frac{\partial p_U^{ND'}}{\partial b'}\left[B'-(1-m_B)B\right]\right]= \beta E_{y'|y}\left(1-h'\right)\left\{u'(c')\left(\left[z_b+q^{ND''}\right](1-m_b)+m_b\right)\right\} \tag{G.7}$$

The left-hand side of equation (G.7) is the benefit of issuing one additional unit of b'. The government can increase its current consumption by $q^{ND'}$ and it also takes into consideration the change in prices due to a change in the probability of default. The right-hand side represents the costs of such issuance. They are given by a decline in future consumption, given the extra coupon and additional principal.

[65] On the quantitative results, the measure of next-period constrained agents (η') is only marginally affected by b' through its effects on prices.

[66] For tractability, I omit here the restriction that imposes that total issuances can be positive only if $q\left(y,b',B',\eta\right)>\underline{q}$.

Taking the first order condition with respect to B':

$$u'(c) \times \left[p_U^{ND} + \frac{\partial p_U^{ND'}}{\partial B'} \left[B' - (1-m_B)B \right] + \frac{\partial q^{ND'}}{\partial B'} \left[b' - (1-m_b)b \right] \right] +$$

$$+\beta \left\{ \frac{\partial E_{y'|y} V\left(y', b', B', \eta'\right)}{\partial B'} + \frac{\partial E_{y'|y} V\left(y', b', B', \eta'\right)}{\partial \eta'} \frac{\partial \eta'}{\partial B'} \right\} + \lambda_2 - \lambda_1 = 0$$

$$\lambda_1 \left[\bar{B} - B' \right] = 0$$

$$\lambda_2 \left[B' - (1-m_B)\, B \right] = 0$$

where λ_i are the Lagrange multipliers. After substituting with (G.1)-(G.4), we get that an interior solution for B' must satisfy:

$$u'(c) \times \left[p_U^{ND'} + \frac{\partial p_U^{ND'}}{\partial B'} \left[B' - (1-m_B)B \right] + \frac{\partial q^{ND'}}{\partial B'} \left[b' - (1-m_b)b \right] \right] =$$

$$\beta E_{y'|y} \left(1-h'\right) \left\{ u'(c') \left(\left[z_B(y') + p_U^{ND''} \right] (1-m_B) + m_B \right) - \frac{\partial V^r \left(y', \tilde{b}\left(B', X\right), B', \eta' \right)}{\partial \eta'} \times \frac{\partial \eta'}{\partial B'} \right\} \tag{G.8}$$

The left-hand side of equation (G.8) represents the benefit of issuing one additional unit of B'. The government can increase its current consumption by $p_U^{ND'}$ and it also takes into consideration the change in prices due to a change in the probability of default. The first term on the right-hand side represents the costs of such issuance. As before, they are given by a decline in consumption, given the extra coupon and additional principal. Finally, the last term on the right-hand side represents an additional benefit in terms of a decline in future liquidity premium (i.e., increase in future prices) given the larger pool of constrained agents. In other words, the cost of an additional unit of B is attenuated by the increase in the next-period pool of constrained investors, that lead to a decrease in the liquidity premium and therefore allows for extra consumption.

Combining (G.7) and (G.8) and using our expression for $\frac{-d\tilde{b}(B',X)}{dB'}$ (equation in 4.4 the main text), we have that:

$$E_{y'|y} \left(1-h'\right) u'(c') \left\{ \frac{-\partial \tilde{b}\left(b', X\right)}{\partial B'} \left(\left[z_b + q^{ND''} \right] (1-m_b) + m_b \right) \right\} =$$

$$E_{y'|y} \left(1-h'\right) \left\{ u'(c') \left(\left[z_B\left(y'\right) + p_U^{ND''} \right] (1-m_B) + m_B \right) - \frac{\partial V^r \left(y', \tilde{b}\left(B', X\right), B', \eta' \right)}{\partial \eta'} \times \frac{\partial \eta'}{\partial B'} \right\} \tag{G.9}$$

which is the same equation as equation 4.7 in section 4.4.